Introduction to Biopolymer Physics

Problem 2.3

Question

A polymer chain with persistence length L_p is confined in a tube of diameter D, so that $L_p \ll D$.

a) Derive the elongation of the polymer along the tube.

b) Derive the scaling relation of the free energy of confinement in terms of the number of links N, D and L_p.

Solution

a) We consider the polymer in the tube, as a chain of blobs with diameter D. Within each blob the polymer behaves as an unconfined chain. From the power law for the polymer radius $R \simeq N^\nu l$ we can derive the number of links g within a blob

$$D \simeq g^\nu l \qquad \to \qquad g \simeq (D/l)^{1/\nu} . \tag{1}$$

Chain of blobs consists of N/g blobs. Hence the extension of the chain in the logitudinal direction is given by

$$R_\| = (N/g)\, D. \tag{2}$$

Using eqn. (1) yields

$$R_\| = Nl\,(D/l)^{\frac{\nu-1}{\nu}} . \tag{3}$$

Note, that Nl is the contour length of the polymer. For a swollen chain with $\nu = 3/5$, we get

$$R_\| = Nl\,(D/l)^{-2/3}. \tag{4}$$

b) We derive an expression for the free energy of confinement F_{conf} based on scaling arguments. Therefore we make certain assumption about its properties. First, the free energy should be an extensive property, i.e. it should be proportional to the length and hence the number of links

$$F_{conf} \sim N. \tag{5}$$

Second, the dimension of the free energy should be that of the thermal energy. We can write

$$F_{conf} \simeq kT\,\phi, \tag{6}$$

where ϕ should be dimensionless and proportional to N in order to satisfy eqn. (5), as kT already has the demanded dimension.

The system has two relevant length scales: the Flory radius R_F and the diameter D of the tube. The function ϕ should depend on R_F and D, but in such a way that it is dimensionless. An obvious choice is a power law

$$\phi = (R_F/D)^n. \tag{7}$$

The exponent n can now be chosen that ϕ is proportional to N. With eqn. (2.45) and $\nu = 3/5$ we obtain

$$F_{conf} \simeq kT\,(R_F/D)^n \simeq kT\,(lN^{3/5}/D)^n. \tag{8}$$

Hence, we get $n = 5/3$ for the exponent. Finally, the scaling law reads

$$F_{conf} \simeq kT\,N\,(l/D)^{5/3}. \tag{9}$$