Introduction to Biopolymer Physics

Problem 2.4

Question

A polymer chain with persistence length L_p is confined in a narrow tube of diameter D, so that $L_p \gg D$.

a) Derive the scaling law for the deflection length and the elongation of the polymer along the tube.

b) Derive the scaling relation of the free energy of confinement in terms of the number of links N, D and L_p.

Solution

a) The undulations of a polymer within the tube are characterized by the deflection length λ and the deflection angle θ. For small θ, the deflection length and angle are related through

$$\theta \simeq \frac{D}{\lambda}. \tag{1}$$

This is a first order approximation, since

$$\frac{D}{\lambda} \simeq \tan\theta = \theta + \mathcal{O}(\theta^3). \tag{2}$$

From eqn. (2.25) in the textbook, we get the mean square bending angle for an elastic worm-like filament

$$\langle \theta^2(s) \rangle = 2\,\frac{s}{L_p}, \tag{3}$$

here s is the filament length. Hence, for the deflection length segement $s = \lambda$, we obtain

$$\langle \theta^2 \rangle = 2\,\frac{\lambda}{L_p} \simeq \left(\frac{D}{\lambda}\right)^2, \tag{4}$$

From the last relation we get the scaling behavior of the deflection length

$$\lambda \simeq D^{2/3} L_p^{1/3}. \tag{5}$$

The elongation of the chain with contour length L in the narrow tube is given by

$$R_\parallel = L\langle \cos\theta \rangle \simeq L\left(1 - \frac{1}{2}\langle \theta^2 \rangle\right), \tag{6}$$

here we made use of the cosine expansion. The relative decrease in length with respect to the fully stretched configuration takes the form

$$\frac{L - R_{\|}}{L} = \frac{1}{2}\langle\theta^2\rangle \simeq (D/L_p)^{2/3}. \tag{7}$$

b) The free energy of confinement should have the dimension of thermal energy kT and it should be extensive in the contour length L. Furthermore the free energy of confinement is proportional to the number of wall induced bends (L/λ). The power law approach

$$F_{conf} \simeq kT\,(L/\lambda)^n \tag{8}$$

with $n = 1$ satisfies all the desired properties. With eqn.(5) we get the demanded expression for the free energy

$$\frac{F_{conf}}{L\,kT} \simeq D^{-2/3}L_p^{-1/3}. \tag{9}$$