Quantum Mechanics: Symbolism of Atomic Measurements. Julian Schwinger. Edited by Berthold-Georg Englert. 496 pp. Springer, New York, 2001. Price: \$54.95

F. Scott Fitzgerald famously remarked that the very rich are different from you and me. Except for their wealth and

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greed, I tend to doubt that this is true. However I'm sure that Julian Schwinger was different from you and me. Like Leibniz, he had an enduring belief that if only one could present things the right way, with the correct notation, it would lead one to see that what is true is really inevitable. This conviction led Leibniz to the modern notation for the Calculus, which was far superior to Newton's. And it led Schwinger to a lifelong search for the perfect way to teach quantum theory.

I remember when I was a graduate student at MIT, and a group of us would ride over to Harvard to watch him teach. He delivered perfect lectures, without notes, without stumbling. But he did not like to be interrupted by questions. And when he was finished, he would be gone in a flash. We were all quite bothered by the puzzle of how someone who seemed to know everything could be so disconcerted by questions he could obviously answer easily. Many years later I got to know him somewhat, and was surprised to discover that he was incredibly shy. However, in one-on-one encounters he was very generous with his knowledge, and at private gatherings could be quite warm and charming.

I used to think it was my private insight that he was a notational genius. If he wrote a summation sign, it could mean integrate for bosons, and differentiate for fermions. Its role was symbolic. He often wrote what he called the "suggestion of an equation," and it all looked so easy in the lecture; but when you got home, you would see that he was multiplying things that didn't commute, or dividing by zero. Then you had to unravel what seemed so simple. If you took careful notes, you would find that he had prepared you for the difficulties, which might have gone right past you in the glibness of the lecture. But he wanted you to see the big picture. The difficulties would come out when you unfolded his equations.

The subtitle of this book spills all the beans. It is "Symbolism of Atomic Measurements," and it represents his belief that if the notation is presented correctly, one will see that the content follows inexorably. The presentation is a refinement of what he was doing forty years ago, and he was always trying to improve it. He starts from Stern-Gerlach experiments with spin, and slowly develops his measurement theory notation, eventually converting it into Dirac notation. The goal is to show you that if you look at things the right way, there is no other way to do it.

Personally, I don't find this to be a good way to teach physics. In a similar way, most modern quantum texts try to get to the "axioms" of quantum theory very quickly, and then show how the subject unfolds from them. But of course the creative process in physics doesn't run this way. In practice, one looks at all the messy phenomena and tries to discern some order in it. I think the ideal teaching method is to present the chaos, and only then the attempts to bring order out of it. Of course, the chaos cannot be *too* deep, or students will get lost. But the experience will prepare them for what the world is like, and how they will be doing research. Physicists don't sit down and formulate axioms before they know what is going on.

That being said, the book is a tour-de-force. Once the groundwork is laid, he goes into subjects with the mathematical virtuosity for which he was famous—not advanced mathematics, but the incredible use of simple mathematics. For example, in teaching the hydrogen atom, he exploits a parallel with the two-dimensional harmonic oscillator. He also connects the solution to the Lenz–Runge vector (without naming it). Then he solves the problem again in para-

bolic coordinates. The 2D oscillator re-emerges in the angular momentum operators. Along the way he offers some remarkable physical insights. But as I said, his mind doesn't work like yours, and you often have to struggle very hard to see what was obvious to him.

At the ends of chapters, he gives a series of problems, some of which supply proofs of points he merely mentioned in the text, but many of which provide new insight into the subject. One cannot afford to pass over the problems. As one example among many, I recently wrote a paper showing that the Galilean transformation cannot be trusted as the non-relativistic limit of the Lorentz transformation, since among other things it conserves mass, and does not take consistently into account that  $E = mc^2$ . Well, sure enough, there is a problem on extending the Galilean transformation to the next order, where it solves the problem, to that order (using some typical Schwingerian mathematical legerdemain). Incidentally, the Galilean transformation, together with Schwinger's Action Principle, plays a major role in determining the non-relativistic Hamiltonian.

For another example, he introduces the magnetic field through the vector potential A, as does everyone else. But, interestingly, he also considers the velocity v as an independent vector. In a homework problem, to introduce spin, he tells you to take the Hamiltonian  $[\sigma \cdot (\mathbf{p} - e\mathbf{A}/c)]^2/2m$ , rather than  $(\mathbf{p} - e\mathbf{A}/c)^2/2m$ . While the two Hamiltonians are equal in the absence of A, the first one gives the correct g-factor (2) for the electron. (It is the nonrelativistic limit of the Dirac equation. Schwinger knows this, but it is one of those things he doesn't bother to tell you. The correct factor just pops out.) Part of the problem is that he wants the notation and the flow of the presentation to lead you to the right answer. And so, while his techniques carefully sidestep the many pitfalls that abound, he feels he would be doing you a disservice to point this out, as it would ruin the inevitability of the result. Unfortunately, pedagogically it would have been a great boon to the presentation.

Almost every topic is treated differently from the standard way. Sometimes this provides great insight, and sometimes he doesn't let you in on the secret. Either way, there are gems throughout the book.

As unique and insightful as the book is, I would never recommend it as a text. Important subjects are left out (such as scattering theory, which is a real shame, since he was the great master) and the Dirac equation (although he has some wonderful stuff on field theory). And the book is really too difficult for most first year graduate students. Rather, it is a wonderful book for a professor to own, like Feyman's lectures, because there is so much to learn from it. But to do so, one has to read it very carefully. It is not a book for idle browsing, and the more you know about a topic, the more you can get from it, because while he does a lot of beautiful things, he doesn't always think it necessary to let you in on why he is doing them. Yes, the man's muse spoke to him privately.

The book was lovingly edited from some UCLA lecture notes, by Berthold-Georg Englert, a longtime student and assistant of Schwinger's, and sometimes Schwinger's voice comes through clearly. Englert has a special insight into this unique and great mind. Of course, it is not the book Schwinger would have given us, as his ideas were being modified until he died, but a lot of it does represent his latest thinking on the subject. We should be thankful for what we do have.

Daniel Greenberger is a professor of Physics at the City College of New York. He has spent most of his career scratching his head over the meaning of quantum mechanics, and reports that other than losing his hair, he hasn't gotten anywhere. As for the success of others, he remains a "skeptomaniac."