# Berthold-Georg Englert

## **Scientific Curriculum** (July 2020)<br>**and List of Publications** (February 2021)

### Scientific Curriculum

(updated in July 2020)

My research activities span more than four decades and diverse topics in theoretical quantum physics. The following remarks on selected papers of mine focus on work in the areas specified by these key words:

1 – Semiclassical theory of many-fermion systems;
2 – Quantum optics;
3 – Classical-quantum boundary;
4 – Complementarity, wave-particle duality, and all that;
5 – Quantum information;
6 – Other work.

**1 – Semiclassical theory of many-fermion systems**

**1a – Electrons in atoms (1981–1993)**

Papers [4–12], summarized and extended in the book [13], deal with refinements of the semiclassical Thomas–Fermi model of atoms to the point where it becomes possible to treat atomic shell structure quantitatively (see [10], in particular). These investigations identify the link between the systematics of the Periodic Table and the properties of the Thomas–Fermi potential, whereby a simple rule is established for the order in which the orbital states become available [8]. A further extension focuses on the 'last' electron [11, 12] and derives the semiclassical prediction for the ionization energy. — The developments up to 1985 are recalled in [201]; see also [207].

**1b – Ultracold trapped atoms (2001 and later)**

Later, these methods (supplemented by the momentum-space considerations in [19, 28, 29, 39, 42]) found an application to cold trapped Fermi gases [96], which deserve further study. When the gas is trapped in a two-dimensional geometry, matters are markedly different from the three-dimensional situation [155]. A systematic study of Airy-averaged gradient corrections for two-dimensional fermion gases — a revival, to some extent, of the three-dimensional investigations in [6] — is conducted in [194], with results that strongly encourage the application to the self-consistent equations for interacting systems.

There is also the little paper [70] on two atoms in a harmonic trap with a contact interactions between them. Judging by the large number of citations, this work is of some importance for experiments with neutral atoms in optical potentials.

Paper [144] concerns cold fermionic atoms trapped in a two-dimensional honeycomb potential and demonstrates the feasibility of implementations with imperfect optical potentials; some of the predictions have been verified in experiments (Esslinger).

**1c – General developments**

In [190] we solve a twenty-five year old mystery about inhomogeneity corrections to the Thomas–Fermi approximation for the kinetic energy of two-dimensional gases of fermions. Contrary to folklore, we show that these corrections are definitely nonzero and evaluate the leading correction in perturbation theory.

Paper [198] introduces a novel method for approximating the single-particle density as a functional of the effective single-particle potential energy, and thus also other functionals, without a gradient expansion. Instead, the density is expressed in terms of unitary evolution operator which is approximated by a factorization of the Suzuki–Trotter kind. A spin-off is the use of this factorization in the fourth-order leapfrog algorithm that is reported in [214].

## 2 – Quantum Optics

### 2a – Master equations and the theory of the micromaser (1993–2006)

Paper [36] introduces the *damping-basis method*, a powerful tool for investigating and solving master equations as they appear in studies of open, driven quantum systems, in particular those of interest in quantum optics. The method enabled us to give a complete analytical solution of the Jaynes–Cummings model with dissipation (a problem that had frustrated many). This is crucial for extending the theoretical treatment of the micromaser to including photon dissipation during the passage of an atom [44]. The damping-basis method is equally useful when dealing with the time-averaged behavior of a periodically pumped micromaser [57, 69, 84], or when studying correlations among emerging atoms [77]. The latter requires calculating the statistics of detector clicks, for which the general formalism is described in paper [43]. The lecture notes [107] are a tutorial introduction into these matters, and [106, 120] deal with further developments.

The review [130] summarizes the literature on cavity quantum electrodynamics both from the experimental and the theoretical viewpoint.

### 2b – Raman transitions (2012–2013)

In the context of our work on a scheme for the robust storage of quantum information (see [156]), we needed to refine the existing methods for treating multi-photon Raman-type transitions that make use of nonresonant intermediate levels. This led us to a systematic improvement over the usual adiabatic-elimination procedure [174] as well as to an alternative approach [175] that does not rely on adiabatic elimination of the intermediate levels.

### 2c – Other matter

Paper [176] deals with quantum correlations as they manifest themselves in the data recorded by lossy detectors that can distinguish between one, two, ore more photons. See also [192].

## 3 – Classical-quantum boundary (1989–1998)

The 1989 paper [16] deals with my unified approach to phase-space functions of various kinds (Wigner, Kirkwood, Glauber). Insights gained then were later useful when giving a rather precise meaning to the classical limit of a quantum-mechanical observable; see papers [25] and [71] in particular.

In [25], general criteria for what can be considered a reasonable phase operator are first established and then applied to a number of plausible candidates. A spin-off is [56] where these ideas helped to analyze a real-world physical experiment.

A puzzling observation is reported in [71]: Unitarily equivalent Hamilton operators can have utterly different classical limits, so that their classical analogs describe quite different physical systems, such as harmonic oscillators with different frequencies. This has an obvious bearing on the question of how to quantize a given classical dynamics.

## 4 – Complementarity, wave-particle duality, and all that (1988–2015)

### 4a – Stern-Gerlach interferometers and Humpty Dumpty

The fully quantum-mechanical treatment of Stern-Gerlach interferometers in papers [14, 15, 18] is a very early study of an atom interferometer, in which we find that it is virtually impossible to recover the original spin coherence when a beam of spin-$\frac{1}{2}$ is split in two and then reunited. This 'Humpty-Dumpty effect' was later demonstrated experimentally (Baudon).

### 4b – Bohr's complementarity and Einstein's wave-particle duality

Lessons learned then were crucial for paper [24] on Bohr's complementarity principle and its link to Heisenberg's uncertainty relation. The more qualitative arguments of [24] received a fully quantitative basis by the work on the quantitative aspects of Einstein's wave-particle duality in paper [67], extended in [68, 73, 76, 81] and summarized in [87]. The crucial step is the derivation of the duality relation in [67] which states the limits on the compromises between the wave character (visibility of interference fringes) of a quantum object and its particle character (path knowledge). Particularly important is the observation that the duality relation is logically independent of Heisenberg's uncertainty relation and its generalizations. Experimental tests of the duality relation have been performed with atoms (Rempe) and photons [76, 80] and proposed for neutrons [78]. An application of these concepts to the situation of coherent double scattering is given in [137].

All these studies deal with two-path interferometers. The generalization to multipath configurations is partly accomplished in [138], where the basic conceptual questions are answered, but further studies are necessary before the picture is complete.

A conjecture in [138] was eventually shown to be wrong, and this triggered renewed interest in entropic measures for path distinguishability and interference strength. Some results are reported in [188] without, however, closing the subject as important problems remain unsolved.

**4c – Mutually unbiased bases**

The concept of mutually unbiased bases is central to these matters — two observables are complementary if their eigenstate bases are unbiased. The review article [151] summarizes what is currently known about such bases and introduces a new problem: How many such bases are there for a periodic continuous degree of freedom? The answer to this question is given in [170]: In full analogy with the other continuous degrees of freedom, there is a continuous set of pairwise unbiased bases for a periodic degree of freedom, too.

There remains the open problem of how many such bases one can have in a six-dimensional Hilbert space, the smallest space in which the standard constructions for maximal sets of unbiased bases does not work. The numerical search that is reported in [159] adds analytical insight in support of the conjecture that there are no more than three pairwise unbiased bases, by establishing the four most distant bases and demonstrating that they are not mutually unbiased.

For mutually unbiased bases in the context of quantum state tomography, see [189].

## 5 – Quantum information (1997 and later)

**5a – Structure of two-qubit states**

Papers [91, 94] and the extensive book chapter [100] report work that began in 1997 and is still not completed in full. It concerns the classification of general two-qubit states — the 'hydrogen molecule' of quantum information theory. These states are specified by fifteen parameters, and the corresponding state space has a very rich structure that is not fully understood even today. Our approach puts much emphasis on geometrical features. In particular, we use two three-component vectors and a $3 \times 3$-component dyadic to specify the two-qubit states. The vectors and the dyadic behave in a transparent manner under unitary transformations, and this facilitates detailed studies enormously.

We are particularly interested in the so-called Lewenstein–Sanpera decompositions which are crucial for determining to which extent the correlations between the two qubits can be mimicked by classical statistics and to which extent they are of a purely quantum nature. We succeeded in finding the optimal decompositions for some important classes of states, but a procedure for general two-qubit states was missing for many years. Finally, owing to the observation that the search for the optimal decomposition can be formulated as a semidefinite program [147], a relatively simple numerical procedure is now at hand that can determine the optimal decomposition for any two-qubit state. It also gives analytical solutions for cases not accessible earlier.

**5b – The Mean King's problem and quantum cryptography**

A different line of research involves an optical realization of Aharonov's 'Mean King's Problem'. It is proposed in paper [93], and [110] reports the experimental realization.

These studies suggested a new method for quantum cryptography and, in particular, for direct secure communication [98, 104]. The latter is fascinating because it enables one to send a confidential message without first establishing a shared key for encryption. This opens up a whole new line of research in quantum information theory. The book chapter [103] summerizes the state of affairs in 2002.

After joining the National University of Singapore, first as a Visiting Professor, then as a permanent faculty member, I got interested in schemes for Tomographic Quantum Cryptography — a very promising field, both for theoretical studies and for proposing experimental implementations of new schemes for quantum key distribution. Paper [117] introduces the general ideas, defines terminology and notation, and explains the strategy for analyzing eavesdropping attacks. In [116], [118], and [121] we describe various results, which also have a bearing on other protocols that are used routinely.

A central problem in these investigations is to find the optimal measurement that fully extracts the accessible information, for which an iterative procedure is described in [124], and an open-source code is made available in [141]; an extension thereof, for the purpose of computing channel capacities, is the open-source code of [162]. The 2007 status of this field is summarized in the book chapter [135], and the optimal measurements for quantum pyramids are given in [142].

A tomographic protocol of particular interest became known as the 'Singapore Protocol', introduced in [126]. It makes use of the minimal qubit tomography of [122] and [125], is more efficient than competing tomographic protocols, and more robust than all other protocols described in the literature. The Singapore protocol has been analyzed in full, but the analysis is not published as yet. The experimental study in [199] covers the Singapore protocol along with others.

Studies of quantum key distribution protocols with partial tomography resulted in paper [136]. A novel protocol with trine states, characterized by a key extraction scheme that is substantially more efficient than the usual scheme, is reported in [148].

**5c – Quantum state tomography; quantum state estimation**

State tomography, in particular of two-qubit states distributed by some source, are the subject matter of papers [143] and [145]. Partly, they extend the single-qubit results of paper [122], but there are also truly novel concepts, such as the tomography with entanglement witnesses and the tomography that exploits witness basis measurements, both introduced in paper [143]. An experiment that demonstrates tomography with witness bases has been performed; see [179] and [181]. Paper [154] is an exhaustive study of highly symmetric generalized measurements for qubit pairs, and paper [161] compares tomography schemes that use product measurements with schemes that are most symmetric.

These "most symmetric" schemes are examples of symmetric informationally complete probability-operator measurements (SIC POMs), much studied in a plethora of theoretical and mathematical publications. Papers [165] and [168] deal with proposals for the actual laboratory implementation of SIC POMs for higher-dimensional quantum degrees of freedom (the qubit case is the subject matter of [122]).

The interpretation of the data acquired by schemes for state tomography or process tomography requires systematic statistical inference. For that purpose, maximum-likelihood estimation is a popular procedure. Paper [157] shows how to supplement it with Jaynes's maximum-entropy principle if the data are incomplete; a comprehensive study of these matter is [166]. The application of these ideas to adaptive process tomography is presented in [164], and [173] deals with related matters, such as the problem of confirming entanglement on the basis of incomplete data.

Maximum-likelihood estimation has a tendency to produce implausible estimators, so that alternatives are worth exploring, among them minimax estimation, which yields conservative estimators. In [167] the research is motivated, a program introduced, and a simple, yet accurate, minimax estimator presented; a somewhat different approach is explored in [172].

All these estimation strategies aim at specifying a best guess for the statistical operator (see also [189] for a particular scenario of incomplete tomographic information). In addition, one needs to attach meaningful error bars. Our answer to this question is in terms of estimator regions, in particular the smallest credible regions; we introduce this research program in [178] and report very encouraging first results. In [191], we show how this strategy is applied to the problem of estimating one or a few properties of the quantum state directly, that is: without estimating the state first. It turns out that the direct estimation is, in fact, preferable as it is more reliable and results in shorter error intervals. The important extension to self-calibrating quantum tomography, where the quantum state and parameters of the apparatus or the transmission line are jointly estimated from the same data, is the topic of [205].

In our approach, the estimators for the quantum state are necessarily quantum states themselves. In [182] we comment on an arXiv posting (later published in Physical Review Letters) that advertises the use of estimators that are not assuredly inside the quantum state space.

**5d – Sampling from the quantum state space**

The methods of [178] and [191] are Bayesian and require the computation of high-dimensional integrals which can only be done with Monte Carlo integration. For this purpose, one needs to sample the quantum state space in accordance with various distributions. Papers [184] and [185] deal with algorithms for generating good samples of this kind, and [195] makes an open-source online repository of tested codes for sampling, and also a selection of large ready-to-use samples, available to the community.

The one-to-one mapping between quantum channels and bi-partite quantum states — the Choi–Jamiołkowski isomorphism — can be exploited for a random sampling of quantum channels by a suitable adaptation of the state-sampling algorithm. This is the subject matter of [206]. In [212] we address the problem of estimating the minimum fidelity of a quantum gate (a basic quantum channel, so to say) directly without full channel tomography.

**5e – Consistency checks**

In the Bayesian reasoning, one encodes pre-measurement knowledge about the situation in the prior density on the state space, and one relies on proper modeling of the situation. The question whether the observed data are typical for the prior can be answered by checks for prior-data conflict, and the model needs to be checked against the data as well. We propose strategies for that in [193], [200], and [204].

Model checking plays a crucial role in [202] which deals with the data from four experiments that probe variants of Bell's inequality. We find that two experiments did not report the correct value for the conversion probability of a short-wavelength photon into two long-wavelength photons, and the correct values could be extracted from the data by careful model checking.

**5f – Other topics**

A possible experimental realization of the trine scheme could make use of the reference-free (RFF) qubits, composed of three spin-$\frac{1}{2}$ particles, that we recently introduced in [139]. RFF qubits can also be used for quantum storage purposes, with each qubit encoded in rotationally invariant states of three ultracold spin-$\frac{1}{2}$ atoms in a two-dimensional lattice. The practical feasibility of this idea is the subject of exciting ongoing theoretical studies; a very long life time is predicted for such qubits [156].

In [186] we describe how a controlled-phase gate can be realized between two neutral atoms of the same kind in a rather simple manner. The scheme owes its simplicity to the use of a single laser pulse that drives the relevant transitions off resonance with the detunings adjusted such that the gate is realized with high fidelity.

The systematic and robust encoding of many qubits in a single continuous quantum degree of freedom is the subject matter of paper [149]. The scheme exploits the observation that the state space of a quantum rotor is equal to the product of the spaces of a genuine qubit and another rotor. This is an invitation to an iteration, which we gladly accept. Coaxial photons that carry orbital angular momentum could be used for laboratory implementation, but no such experiments have been realized as yet.

Another line of research aims at combining the advantages of quantum computation by unitary evolution with those of quantum computation by measurement into a hybrid scheme. This can be done and is potentially useful, indeed; see [153]. During these studies, a lesson was learned about quantum search algorithms, which is the subject matter of [158].

Paper [208] contributes to the discussion about overcoming the resolution limits of classical optics withy the aid of quantum aspects of light. We conclude that some claims do not survive scrutiny.

## 6 – Other work

### 6a – Research

The 1998 paper [74] is a parody on the hype about quantum computation in those days. It is probably my most-read paper.

Paper [146] deals with an old subject that is still not completely settled: the transmission of waves through a linear random stack of partially transmitting mirrors. We established a recurrence relation that enabled us to improve on earlier treatments and derive strict upper and lower bounds (both exponential in the number of slabs) on the average transmission probability. The finishing touch is put on in [177], where that recurrence relation is solved explicitly in terms of Legendre functions and a number of analytical results are derived.

Owing to fortunate circumstances, I was involved in experimental studies of the spin Hall effect in platinum, carried out at IMRE [140, 150]. While being off my usual track, this activity was eventually rather rewarding after solid data demonstrated a giant spin Hall conductivity [150], about a factor of 100 larger than any conductivities measured earlier by others.

There is also the colloquium on quantum theory [180], in which I assure the reader that quantum theory is a well-defined local theory with no unsolved foundational problems. The alleged great mysteries result from one misunderstanding or another.

Paper [35] explains why the trajectories of Bohmian mechanics cannot be regarded as stating the historical past of a quantum particle. Another, and very different, attempts at ascribing a past to a quantum particle is criticized in [196]; see also [203]. Both cases have in common that, when one monitors the particle's path through and interferometer, the observed past is at variance with what these proposals say. The experiment reported in [197] was triggered by the theoretical study in [196].

An appendix in [151] showed a connection between the Hilbert-space formalism of quantum theory and number theory and put on record a curious multiplicative function, found by serendipity. As shown in [211] it is a particular representative of two large families of unusual multiplicative functions.

### 6b – Books

The book [13] records the sequence of lectures I gave in 1985 on papers [4–10] and contains other material as well.

I put Julian Schwinger's notes on quantum mechanics [95] into print. This book is much more a posthumous publication by him than a text by me.

The three companion books [131–133] are the lecture notes for my quantum-mechanics courses at NUS; my notes on classical electrodynamics are book [183], the notes on classical mechanics are book [187], and the notes on statistical mechanics are book [214].

Together with others, I co-edited the three books [134, 152, 160]. Two of them grew out of workshops held in Singapore. The editing of book [210] did not involve others; the book includes [209] which is of historical interest.

**List of Publications***
(updated in February 2021)

1. BGE, J. Karkowski, and J. M. Rayski, Jr.
*Conditions on Classical Sources for a Quantum Scalar Field with Higher Order Derivatives*
Physics Letters **83B**, 399–402 (1979).
2. *Quantization of the Radiation-Damped Harmonic Oscillator*
Annals of Physics **129**, 1–21 (1980).
3. W. Dittrich and BGE
*One-Loop Thermal Corrections in the Gross-Neveu Model*
Nuclear Physics **B179**, 85–105 (1981).
4. BGE and J. Schwinger
*Thomas-Fermi revisited: The outer regions of the atom*
Physical Review A **26**, 2322–2329 (1982).
5. BGE and J. Schwinger
*Statistical atom: Handling the strongly bound electrons*
Physical Review A **29**, 2331–2338 (1984).
6. BGE and J. Schwinger
*Statistical atom: Some quantum improvements*
Physical Review A **29**, 2339–2352 (1984).
7. BGE and J. Schwinger
*New statistical atom: A numerical study*
Physical Review A **29**, 2353–2363 (1984).
8. BGE and J. Schwinger
*Semiclassical atom*
Physical Review A **32**, 26–35 (1985).
9. BGE and J. Schwinger
*Linear degeneracy in the semiclassical atom*
Physical Review A **32**, 36–46 (1985).
10. BGE and J. Schwinger
*Atomic-binding-energy oscillations*
Physical Review A **32**, 47–63 (1985).

— J. Schwinger and BGE
*The statistical atom*
unpublished (1985).†

11. *Weakly ionized Thomas-Fermi atoms*
Physical Review A **33**, 2146–2147 (1986).
12. *Statistical Atom: Ionization Energies*
Zeitschrift für Naturforschung **42a**, 825–834 (1987).
13. *Semiclassical Theory of Atoms*
Lecture Notes in Physics, Vol. 300
(Springer-Verlag, Berlin and Heidelberg, 1988)
ISBN 3–540–19204–2.
14. BGE, J. Schwinger, and M. O. Scully
*Is Spin Coherence like Humpty-Dumpty? I. Simplified Treatment*
Foundations of Physics **18**, 1045–1056 (1988)
(invited contribution to a Festschrift for David Bohm).
15. J. Schwinger, M. O. Scully, and BGE
*Is spin coherence like Humpty-Dumpty? II. General theory*
Zeitschrift für Physik **D10**, 135–144 (1988);
reprinted in the Proceedings of the Eleventh International Conference on Atomic Physics
(11th ICAP), Paris 1988 (World Scientific, Singapore 1989, edited by S. Haroche *et al.*),
pp. 37–62.

*Papers 35(a), 36, 59, 70, 74, 78, 97, 99, 102, 105, 113, 115, 118(b), 122, 135, 143, 144, 151, 157, 168, 175, 178, 180, and 181 are fully or partly reprinted in *Quantum Paths*, edited by Rui Han and Hui Khoon Ng (World Scientific Publishing Company Co., Singapore 2015).

†This paper was an invited contribution to Physikalische Blätter, but the editor did not like the article and did not put it into print. It got eventually published in the proceedings of the Schwinger Centennial Conference; see 201 and 210.

16. *On the operator bases underlying Wigner's, Kirkwood's and Glauber's phase space functions*
Journal of Physics A: Mathematical and General **22**, 625–640 (1989).
17. BGE and J. Schwinger
*Thomas-Fermi Quantization, Classical Orbits, and the Systematics of the Periodic Table*
Proceedings of the International Conference on Classical Dynamics in Atomic and Molecular Physics (CDAMP '88), Brioni 1988 (World Scientific, Singapore 1989, edited by T. Grozdanov *et al.*), pp. 371–387.
18. M. O. Scully, BGE, and J. Schwinger
*Spin coherence and Humpty-Dumpty. III. The effects of observation*
Physical Review A **40**, 1775–1784 (1989).
19. K. Buchwald and BGE
*Thomas-Fermi-Scott model: Momentum-space density*
Physical Review A **40**, 2738–2741 (1989).
20. BGE and M. O. Scully
*Good and Bad Welcher Weg Detectors*
Proceedings of the NATO Conference on New Frontiers in Quantum Electrodynamics and Quantum Optics, Istanbul 1989 (Plenum Press, New York 1990, edited by A. O. Barut, NATO ASI Series, Vol. B232), pp. 507–512.
21. BGE, J. Schwinger, and M. O. Scully
*Center-of-Mass Motion of Masing Atoms*
Proceedings of the NATO Conference on New Frontiers in Quantum Electrodynamics and Quantum Optics, Istanbul 1989 (Plenum Press, New York 1990, edited by A. O. Barut, NATO ASI Series, Vol. B232), pp. 513–519.
22. *Spin Coherence in Stern-Gerlach Interferometers*
Proceedings of the NATO Conference on New Frontiers in Quantum Electrodynamics and Quantum Optics, Istanbul 1989 (Plenum Press, New York 1990, edited by A. O. Barut, NATO ASI Series, Vol. B232), pp. 521–530.
23. BGE, J. Schwinger, A. O. Barut, and M. O. Scully
*Reflecting Slow Atoms from a Micromaser Field*
Europhysics Letters **14**, 25–31 (1991).
24. M. O. Scully, BGE, and H. Walther
*Quantum optical tests of complementarity*
Nature **351**, 111–116 (1991).
25. J. Bergou and BGE
*Operators of the Phase. Fundamentals*
Annals of Physics **209**, 479–505 (1991).
26. H.-J. Briegel, BGE, M. Michaelis, and G. Süssmann
*Über die Wurzel aus der Klein-Gordon-Gleichung als Schrödingergleichung eines relativistischen Spin-*0*-Teilchens*
Zeitschrift für Naturforschung **46a**, 925–932 (1991).
27. H.-J. Briegel, BGE, and G. Süssmann
*Canonical Quantization of the Classical Hamiltonian for a Relativistic Spin-*0 *Particle*
Zeitschrift für Naturforschung **46a**, 933–938 (1991).
28. *Energy functionals and the Thomas-Fermi model in momentum space*
Physical Review A **45**, 127–134 (1992).
29. M. Cinal and BGE
*Thomas-Fermi-Scott model in momentum space*
Physical Review A **45**, 135–139 (1992).
30. BGE, H. Fearn, M. O. Scully, and H. Walther
*An atomic-beam quantum-eraser gedanken experiment*
Proceedings of the NATO Advanced Research Workshop on Quantum Measurements in Optics, Cortina d'Ampezzo 1991 (Plenum Press, New York 1992, edited by P. Tombesi and D. F. Walls), pp. 55–62.
31. BGE, H. Walther, and M. O. Scully
*Quantum Optical Ramsey Fringes and Complementarity*
Applied Physics **B54**, 366–368 (1992).

32. *Complementarity*
Proceedings of the Santa Fe 1991 Workshop on the Foundations of Quantum Mechanics (World Scientific, Singapore 1992, edited by T. D. Black *et al.*), pp. 181–192.
33. BGE und H. Walther
*Komplementarität in der Quantenmechanik*
Physik in unserer Zeit **23**, 213–220 (1992).
34. *Time Reversal Symmetry and Humpty-Dumpty*
Zeitschrift für Naturforschung **52a**, 13-14 (1997);
Proceedings of the workshop in honor of E. C. G. Sudarshan, Austin 1991, edited by BGE and G. Süssmann.
35. BGE, G. Süssmann, M. O. Scully, and H. Walther
(a) *Surrealistic Bohm Trajectories*
Zeitschrift für Naturforschung **47a**, 1175–1186 (1992).
(b) *Reply to Comment on 'Surrealistic Bohm Trajectories'*
Zeitschrift für Naturforschung **48a**, 1263–1264 (1993).
36. H.-J. Briegel and BGE
*Quantum optical master equations: The use of damping bases*
Physical Review A **47**, 3311–3329 (1993).
— *Sonolumineszenz – Casimir-Licht aus einer Wasserblase?*
Physik in unserer Zeit **24**, 100–101 (1993).
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*Quantum optical master equations: The one-atom laser*
Physical Review A **48**, 732–738 (1993).
38. BGE, N. Sterpi, and H. Walther
*Parity states in the one-atom maser*
Optics Communications **100**, 526–535 (1993).
39. M. Cinal and BGE
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Physical Review A **48**, 1893–1902 (1993).
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41. BGE, H. Fearn, M. O. Scully, and H. Walther
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— BGE, M. O. Scully, and H. Walther
*One-atom maser: Recoilfree photon emission*
unpublished (1993).
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*Komplementarność*
Delta, November 1993, pp. 1–4.
42. B. Rohwedder and BGE
*Semiclassical quantization in momentum space*
Physical Review A **49**, 2340–2346 (1994).
43. H.-J. Briegel, BGE, N. Sterpi, and H. Walther
*One-atom maser: Statistics of detector clicks*
Physical Review A **49**, 2962–2985 (1994).
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Physical Review A **49**, 5019–5041 (1994).

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*Quantum-optical master equations: An interaction picture*
Physical Review A **50**, 2667–2679 (1994).

46. E. Wehner, R. Seno, N. Sterpi, BGE, and H. Walther
*Atom pairs in the micromaser*
Optics Communications **110**, 655–669 (1994).

47. M. Battocletti and BGE
*Reflecting slow atoms from a damped resonator*
Journal de Physique II **4**, 1939–1953 (1994).

48. BGE, C. Miniatura, and J. Baudon
*Least-bias description of atomic beams*
Journal de Physique II **4**, 2043–2059 (1994).

49. BGE, M. O. Scully and H. Walther
*The Duality in Matter and Light*
Scientific American **271**(6) (December 1994), pp. 56–61 (international edition); pp. 86–92 (US edition).

50. *Elements of micromaser physics*
eprint arXiv:quant-ph/0203052
Written for the Proceedings of the 19th International Nathiagali Summer College on Physics and Contemporary Needs, Nathiagali 1994 (edited by S. A. Ahmad and S. M. Farooqi for Pak Book Cooperation), which never appeared in print.

51. K. Wódkiewicz and BGE
*Quantum trigonometry and phase-space propensity*
in: Quantization, Coherent States, and Complex Structures (Proceedings of the XIIIth Workshop on Geometric Methods in Physics, Białowieża 1994) (Plenum Press, New York 1995, edited by J.-P. Antoine *et al.*), pp. 243–248.

52. BGE and K. Wódkiewicz
*Intrinsic and operational observables in quantum mechanics*
eprint arXiv:quant-ph/9502013
Physical Review A **51**, R2661–R2664 (1995).

53. *Driven Systems with One Bound State*
Letters in Mathematical Physics **34**, 239–248 (1995)
(invited contribution to the memorial issue for Julian Schwinger).

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## Bibliometry

On 1 August 2020, the Web of Science (Publons) counted 175 articles with 6,375 citations (most recent papers are not included), and found an h-index of 39. The following table displays the citation data for the ten most-cited papers. (The citation counts in parentheses are those of Google Scholar, with h = 51.)

| Paper | citations | since | Paper | citations | since |
|---|---|---|---|---|---|
| 24 | 597 (1,025) | May 1991 | 151 | 306 (510) | June 2010 |
| 67 | 487 (822) | September 1996 | 36 | 183 (273) | April 1993 |
| 70 | 459 (748) | April 1998 | 23 | 140 (171) | January 1991 |
| 130 | 454 (831) | May 2006 | 122 | 131 (223) | November 2004 |
| 104 | 354 (518) | March 2002 | 76 | 111 (183) | December 1999 |

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