Problem 1 (10 points)
Atoms that have been pre-selected as "+ in z" are successively passed through first a "+ in x" selector, then a "$-$ in z" selector.
Which fraction of the atoms is let through?

Problem 2 (15 points)
A source emits atoms such that each of them is either "+ in x" or "+ in z", choosing randomly between these options, with equal chances for both. What are the probabilities for

(a) finding the next atom as "+ in x" or "$-$ in x";
(b) finding the next atom as "+ in y" or "$-$ in y";
(c) finding the next atom as "+ in z" or "$-$ in z",

when the respective experiments are performed?

Problem 3 (20 points)
Atoms are prepared such that their magnetic properties are described by the ket

$$|{\uparrow_z}\rangle\frac{2}{3}+|{\downarrow_z}\rangle\frac{1+2\mathrm{i}}{3} \mathrel{\widehat{=}} \frac{1}{3}\begin{pmatrix} 2 \\ 1+2\mathrm{i} \end{pmatrix}.$$

What are the probabilities for

(a) finding the next atom as "+ in x" or "$-$ in x";
(b) finding the next atom as "+ in y" or "$-$ in y";
(c) finding the next atom as "+ in z" or "$-$ in z",

when the respective experiments are performed?

Problem 4 (15 points)
Express the operator product

$$(\sigma_x \cos\phi + \sigma_z \sin\phi)(\sigma_z \cos\phi - \sigma_x \sin\phi)$$

as a linear function of $\vec{\sigma}$, whereby ϕ is an arbitrary angle parameter.

Problem 5 (20 points)
Express the operator

$$A = |{\uparrow_x}\rangle\langle{\uparrow_z}| + |{\downarrow_x}\rangle\langle{\downarrow_z}|$$

as a linear function of $\vec{\sigma}$. What is A^2?

Problem 6 (20 points)
Consider n pairs of kets, the k-th pair denoted by $|a_k\rangle$ and $|b_k\rangle$, that are jointly defined by

$$|a_k\rangle = |{\uparrow_z}\rangle u_k^* + |{\downarrow_z}\rangle v_k\,, \qquad |b_k\rangle = |{\uparrow_z}\rangle v_k^* - |{\downarrow_z}\rangle u_k\,,$$

for $k = 1, 2, \ldots, n$, whereby the amplitudes u_k and v_k are arbitrary complex numbers. Then,

(a) how large are the probabilities $|\langle a_k|b_k\rangle|^2$?
(b) how are the probability amplitudes $\langle a_j|a_k\rangle$ and $\langle b_j|b_k\rangle$ related to each other?