PART I

Answer ALL of the following FIVE short questions.

Short Question S1 (8 marks)
A source emits magnetic silver atoms such that each of them is either $\uparrow_x$ or $\downarrow_z$, choosing randomly between these options, with equal chances for both. These atoms are successively passed through

first a $\uparrow_x$-selector, then a $\downarrow_z$-selector.

Which fraction of the atoms is let through?

Short Question S2 (9 marks)
Silver atoms are prepared such that their magnetic properties are described by the ket

$$|\uparrow_z\rangle \frac{1-\mathrm{i}}{2} + |\downarrow_z\rangle \frac{\sqrt{2}}{2} \mathrel{\widehat{=}} \frac{1}{2}\begin{pmatrix} 1-\mathrm{i} \\ \sqrt{2} \end{pmatrix}.$$

What are the probabilities for

(a) finding the next atom as $\uparrow_x$ if σ_x is measured?
(b) finding the next atom as $\uparrow_y$ if σ_y is measured?
(c) finding the next atom as $\uparrow_z$ if σ_z is measured?

Short Question S3 (8 marks)
Show that

$$f(\sigma_x)\sigma_z = \sigma_z f(-\sigma_x)$$

holds for the products of Pauli operator σ_z with <u>any</u> function of Pauli operator σ_x.

Short Question S4 (6 marks)
State the differential operator that represents $XP + PX$ in the context of position wave functions.

Short Question S5 (9 marks)
A certain Hamilton operator H is represented by the 3×3 matrix

$$\mathcal{H} = \begin{pmatrix} E & 0 & E \\ 0 & -2E & 0 \\ E & 0 & E \end{pmatrix} \text{ and the state ket } |\;\rangle \text{ by the column } \psi = \begin{pmatrix} 2/3 \\ 1/3 \\ -2/3 \end{pmatrix}.$$

Calculate the expectation value $\langle H \rangle$ of the Hamilton operator and its spread δH.

PART II

Answer ANY TWO OF the following three long questions.

Long Question L1 (30 marks)
A certain Hamilton operator H is represented by the 3×3 matrix

$$\mathcal{H} = \hbar\omega \begin{pmatrix} 2 & 0 & \mathrm{i} \\ 0 & 2 & 0 \\ -\mathrm{i} & 0 & 2 \end{pmatrix}, \text{ and the initial column } \psi_0 \equiv \psi(t=0) = \begin{pmatrix} 2\mathrm{i}/3 \\ 1/3 \\ 2/3 \end{pmatrix}$$

represents the state ket $|\ \rangle$.

(a) Determine the eigenvalues, eigencolumns, and eigenrows of $\mathcal{H}$.
(b) If one measured H at $t = 0$, what would be the possible measurement results?
(c) With which probabilities would they occur?
(d) Calculate $\psi(t)$.
(e) Calculate the "probability for no change after the elapse of time t", that is: $p(t) = |{\psi_0}^\dagger \psi(t)|^2$.

Long Question L2 (30 marks)
The position wave function of an atom is $\psi(x) = C\mathrm{e}^{-\frac{1}{4}(x/a)^2 + \mathrm{i}kx}$ with $C > 0$, $a > 0$, and k real.

(a) Express the normalization constant C in terms of a and k.
(b) Find the corresponding momentum wave function $\psi(p)$.
(c) What are the expectation values of X and P?
(d) What are their spreads?
(e) Evaluate the expectation value $\langle(XP+PX)\rangle$, and compare it with $2\langle X\rangle\langle P\rangle$.

Long Question L3 (30 marks)
The Hamilton operator for an atom moving along the x axis under the influence of the constant force F is $H = P^2/(2M) - FX$. At $t = 0$ we have the expectation values $\langle X(0)\rangle = 0$, $\langle X(0)^2\rangle = (\delta X)^2$ and $\langle P(0)\rangle = 0$, $\langle P(0)^2\rangle = (\delta P)^2$ as well as $\langle\, [X(0)P(0) + P(0)X(0)]\,\rangle = 0$.

(a) Can we chose $\delta X > 0$ and $\delta P > 0$ arbitrarily and independently, or are the values restricted? If applicable, state the restriction.
(b) Solve the Heisenberg equations of motion, that is: express $X(t)$ and $P(t)$ in terms of $X(0)$ and $P(0)$.
(c) Find the time-dependent spreads $\delta X(t)$ and $\delta P(t)$, and comment briefly on your results.