Problem 1 (15 marks)
Your friend told you that $u(x, y) = \cos x \cosh y$ is the real part of an entire function. Determine the imaginary part $v(x, y)$. Then write the function compactly.

Problem 2 (15 marks)
The values $f(z)$ of a certain entire function are real for all real values of z. Show that $f(z^*) = f(z)^*$. [Hint: Consider the Taylor expansion at $z = 0$.]

Problem 3 (20 marks)
Evaluate

$$\int_{z_1}^{z_4} \mathrm{d}z \, |z|^2$$

along this path:
(1) first follow the straight line from $z_1 = -1$ to $z_2 = -r$ with $r > 0$;
(2) then follow either one of the two half-circles connecting $z_2 = -r$ with $z_3 = r$;
(3) finally follow the straight line from $z_3 = r$ to $z_4 = 1$.

Problem 4 (20 marks)
For which values of z has

$$f(z) = \frac{\mathrm{e}^{-\mathrm{i}z}}{(z-2)(z^2-4)}$$

singularities? What are the residues of $f(z)$ at those singularities?

Problem 5 (30 marks)
For real $t \neq 0$, evaluate

$$\int_0^{2\pi} \frac{\mathrm{d}\varphi}{2\pi} \frac{\cosh t}{\sinh t + \mathrm{i} \sin \varphi}$$

with the aid of the residue method.