Problem 1 (25 marks)
Statistical operator ρ is a blend of three projectors,

$$\rho = |1\rangle w_1 \langle 1| + |2\rangle w_2 \langle 2| + |3\rangle w_3 \langle 3| \,.$$

The respective kets can be represented by two-component columns, and ρ by a corresponding 2×2 matrix,

$$|1\rangle \mathrel{\widehat{=}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |2\rangle \mathrel{\widehat{=}} \frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix}, \qquad |3\rangle \mathrel{\widehat{=}} \frac{1}{5}\begin{pmatrix} 3 \\ -4 \end{pmatrix}, \qquad \rho \mathrel{\widehat{=}} \frac{1}{20}\begin{pmatrix} 12 & 3 \\ 3 & 8 \end{pmatrix}.$$

Determine the values of the weights w_1, w_2, and w_3.

Problem 2 (25 marks)
Operator F is a function of position operator X and momentum operator P that is specified by

$$\langle x|F|p\rangle = \mathrm{e}^{-\,(a|x| + b|p|)\,/\hbar},$$

where a and b are positive constants. What is $\langle p|F|x\rangle$?

Problem 3 (25 marks)
Operator Λ is a function of position operator X and momentum operator P that is given by

$$\Lambda = \frac{1}{\hbar}(p_0 X + x_0 P) \qquad \text{with } x_0 p_0 = \hbar \,.$$

Its eigenvalues λ comprise all real numbers, $\Lambda|\lambda\rangle = |\lambda\rangle\lambda$. Find the position wave function $\langle x|\lambda\rangle$ to eigenket $|\lambda\rangle$ with the convention

$$\langle x|\lambda\rangle = (\text{a } x_0 \text{ dependent pre-factor}) \times (\text{a function of } x/x_0 - \lambda),$$

with the pre-factor determined by the normalization condition $\langle\lambda|\lambda'\rangle = \delta(\lambda - \lambda')$.

Problem 4 (25 marks)
Motion along the x axis with a constant force F acting: position operator X, momentum operator P, Hamilton operator $H = \dfrac{P^2}{2M} - FX$.

(a) State the Heisenberg equations of motion for $P(t)$ and $X(t)$, solve them, and evaluate the commutator $[X(t), X(t_0)]$.

(b) Then find the three right-hand sides in

$$\mathrm{i}\hbar\frac{\partial}{\partial x}\log\langle x,t|x',t_0\rangle = \textbf{?}\,, \qquad \mathrm{i}\hbar\frac{\partial}{\partial x'}\log\langle x,t|x',t_0\rangle = \textbf{?}\,,$$

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\log\langle x,t|x',t_0\rangle = \textbf{?}$$

in terms of x, x', and $T = t - t_0$.
[As always, $\log y$ denotes the natural logarithm of y.]