Problem 1 (30 marks)
A group has the neutral element $\mathbf{1}$ and five more elements A, B, C, D, E. Some compositions, written as products, are $AA = B$, $AB = CC = \mathbf{1}$, $CA = BC = D$, and $AC = CB = E$. Fill the gaps in the composition table

	$\mathbf{1}$	A	B	C	D	E
$\mathbf{1}$	$\mathbf{1}$	A	B	C	D	E
A	A	B	$\mathbf{1}$	E		
B	B			D		
C	C	D	E	$\mathbf{1}$		
D	D					
E	E					

Find one subgroup with three elements. Are there also subgroups with two elements? How many?

Problem 2 (20 marks)
The elements $\boldsymbol{g} = (a, b, c)$ of G are ordered triplets of real numbers a, b, and c, one element for each triplet, whereby no restrictions are imposed on a, b, or c. Their compositions, written as products, are defined by the rule

$$\boldsymbol{g}_1\boldsymbol{g}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2 + a_1 b_2) \,.$$

Show that, with this composition rule, G is a group. Is it an Abelian group?

Problem 3 (15 marks)
Evaluate the integral

$$\int_0^\infty \mathrm{d}t\, \frac{\cos(at) - \cos(bt)}{t} \qquad (a, b > 0)$$

with the aid of a Laplace transform.

Problem 4 (15 marks)
Evaluate the convolution integral

$$\int_0^t \mathrm{d}t'\, J_0(t - t')J_0(t')$$

by exploiting the fact that the Laplace transform of $J_0(t)$ is $\dfrac{1}{\sqrt{1+s^2}}$.

Problem 5 (20 marks)
Function $f(t)$ is periodic with period T, that is: $f(t + T) = f(t)$. Show that its Laplace transform $F(s)$ is given by

$$F(s) = \frac{1}{1 - \mathrm{e}^{-sT}} \int_0^T \mathrm{d}t\, \mathrm{e}^{-st} f(t) \,.$$

What is the analogous statement for a function $g(t)$ that is "anti-periodic" in the sense of $g(t + T) = -g(t)$?