Problem 1 (15 points)
Consider the situation of sections 2-1 to 2-3 in the notes, and imagine that you have found the scattering operator $S(T)$ for the given H_0 and H_1. Now you replace H_1 by $\widetilde{H}_1 = W^\dagger H_1 W$ where W is a unitary operator that commutes with H_0. How is the resulting new scattering operator $\widetilde{S}(T)$ related to $S(T)$?

Problem 2 (25 points)
A harmonic oscillator (mass M, circular frequency ω, position operator X, momentum operator P) experiences a time-dependent force $F(t)$ that increases *slowly* from $F(t<0) = 0$ to $F(t>T) = F_\infty$. Before the force is acting, the oscillator is in its ground state. What is the probability of finding the oscillator in this no-force ground state at time $t = T$?

Problem 3 (20 points)
The Hamilton operator $H = \hbar\omega A^\dagger A + \displaystyle\int \mathrm{d}\lambda\, \hbar\omega_\lambda a_\lambda^\dagger a_\lambda - \int \mathrm{d}\lambda\, \hbar\Omega_\lambda\big(A^\dagger a_\lambda + a_\lambda^\dagger A\big)$ describes a "central oscillator" (ladder operators $A^\dagger, A$) in interaction with a large number of "bath oscillators" (labeled by λ; ladder operators $a_\lambda, a_\lambda^\dagger$ with commutators $[a_\lambda, a_{\lambda'}^\dagger] = \delta(\lambda - \lambda')$ and $[a_\lambda, a_{\lambda'}] = 0$) whereby the respective coupling strength $\hbar\Omega_\lambda$ between the central oscillator and the λth bath oscillator is very weak. Derive the transition rate from "central operator in its first excited state and all bath operators in their ground states" to "central operator in its ground state and either one of the bath operators in its first excited state" with the aid of Fermi's golden rule.

Problem 4 (25=15+10 points)
An unstable molecule is in the angular momentum state with $j = \frac{1}{2}$ and $m = \frac{1}{2}$ and decays spontaneously into two fragments with $j_1 = 1$ and $j_2 = \frac{1}{2}$, respectively.

(a) What are the probabilities that the first fragment is found with $m_1 = 1$, $m_1 = 0$, or $m_1 = -1$?

(b) What are the probabilities that the second fragment is found with $m_2 = \frac{1}{2}$ or $m_2 = -\frac{1}{2}$?

Problem 5 (15=5+10 points)
A system is composed of three spin-$\frac{1}{2}$ constituents. As usual, we denote by $|{\uparrow\uparrow\downarrow}\rangle$, for instance, the ket for the state where the first constituent has the spin up in the z direction, the second spin is up as well, and the third spin is down. The states described by $|{\uparrow\uparrow\downarrow}\rangle$, $|{\uparrow\downarrow\uparrow}\rangle$, $|{\downarrow\uparrow\uparrow}\rangle$ have $m_s = \frac{1}{2}$ for the total spin.

(a) Which superposition of these three states has total spin $s = \frac{3}{2}$?

(b) Which superposition<u>s</u> of these three states have total spin $s = \frac{1}{2}$? State one such superposition, then another one that is orthogonal to the first, then yet another that is orthogonal to the first and the second, and so forth until you cannot find a next one.