Problem 1 (25 marks)
The two 2×2 matrices

$$S = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad R = \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$$

are elements of a matrix group with just a few group elements. By considering S^{-1}, R^{-1}, S^2, SR, RS, R^2, ..., find the other group elements. Is the group abelian? If it isn't, identify the abelian subgroups.

Problem 2 (25 marks)
The set G consists of all complex 2×2 matrices $M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$ whose matrix elements are restricted by the relations

$$|M_{11}|^2 = |M_{22}|^2 = 1 + |M_{12}|^2 = 1 + |M_{21}|^2, \qquad M_{21}^* M_{11} = M_{12} M_{22}^*.$$

Demonstrate that $M_{11}^* M_{12} = M_{22} M_{21}^*$, and then show that G is a group with matrix multiplication as the group composition law.

Problem 3 (25 marks)
Function $f(t)$ obeys the differential equation

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2} - 3\frac{\mathrm{d}}{\mathrm{d}t} + 2\right) f(t) = 2$$

and has the $t = 0$ values $f(0) = 1$ and $\dfrac{\mathrm{d}f}{\mathrm{d}t}(0) = 2$. First find the Laplace transform $F(s)$ of $f(t)$, and then $f(t)$ itself.

Problem 4 (25 marks)
Consider the family of functions $f_1(t), f_2(t), \ldots$ that are defined by

$$f_n(t) = \frac{1}{n!}\,\frac{n}{T}\left(\frac{nt}{T}\right)^n \mathrm{e}^{-nt/T} \qquad \text{with } T > 0\,.$$

In order to determine the $n \to \infty$ limit of $f_n(t)$, first find the Laplace transform $F_n(s)$ of $f_n(t)$, then evaluate $F_\infty(s) = \lim_{n\to\infty} F_n(s)$, and finally establish $f_\infty(t) = \lim_{n\to\infty} f_n(t)$.