Problem 1 (40 marks)
A "full-wave antenna" is modeled by the electric current density

$$\vec{j}(\vec{r},t) = \begin{cases} \vec{e}_z I\cos(\omega t)\delta(x)\delta(y)\sin(\omega z/c) & \text{for } |\omega z/c| \leq \pi\,, \\ 0 & \text{for } |\omega z/c| \geq \pi\,, \end{cases}$$

where the current I and the radial frequency ω are given constants. Find the angular distribution of the radiated power, averaged over one period of the oscillation, and discuss its properties.

Problem 2 (30 marks)
At $t = 0$, point charges e and $-e$ are created at $\vec{r} = 0$. Charge e stays at rest while charge $-e$ moves with constant velocity $\vec{v}$. Derive the distribution in frequency and angle of the emitted radiation. Why does your result exhibit an unphysical feature, and what is its origin? Sketch the angular distribution for $v \ll c$ and for $v \lesssim c$.

Problem 3 (30 marks)
In lecture, the angular distribution of synchrotron radiation emitted into the mth harmonic in parallel and perpendicular polarization was found to be given by

$$\left(\frac{\mathrm{d}P_m}{\mathrm{d}\Omega}\right)_{\parallel} = \frac{\omega_0}{2\pi}\frac{e^2}{R}\beta^3 m^2 \left[J_m'\big(m\beta\sin\theta\big)\right]^2\,,$$

$$\left(\frac{\mathrm{d}P_m}{\mathrm{d}\Omega}\right)_{\perp} = \frac{\omega_0}{2\pi}\frac{e^2}{R}\beta^3 m^2 \left[\frac{J_m\big(m\beta\sin\theta\big)}{\beta\tan\theta}\right]^2\,,$$

with $\beta = v/c$. Show that the total radiated powers,

$$P_{\parallel} = \sum_{m=1}^{\infty}\int \mathrm{d}\Omega \left(\frac{\mathrm{d}P_m}{\mathrm{d}\Omega}\right)_{\parallel}\,, \qquad P_{\perp} = \sum_{m=1}^{\infty}\int \mathrm{d}\Omega \left(\frac{\mathrm{d}P_m}{\mathrm{d}\Omega}\right)_{\perp}\,,$$

are such that $P_{\parallel}/P_{\perp} = 3$ for $\beta \ll 1$.

Hints: (1) You do not need to find $P_{\parallel}$ and $P_{\perp}$ for arbitrary values of β.
(2) You may want to establish the form of $J_m(z)$ for $|z| \ll 1$ before you do anything else.