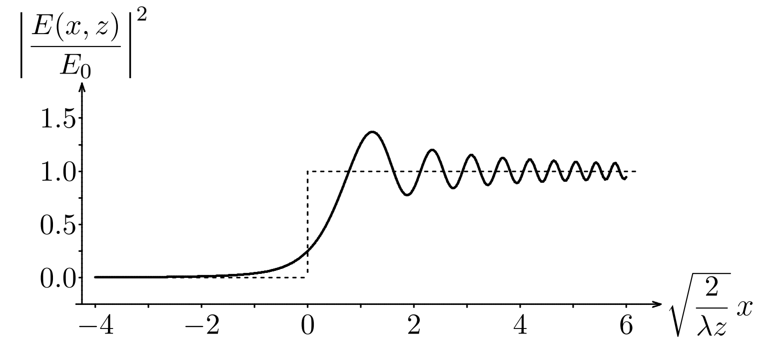
Problem 1 (30 marks)
Proceed from the angular distribution of synchrotron radiation, derived in Section 9.8, and re-derive the total radiated power of (9.3.15).

Problem 2 (30 marks)
When a dielectric ball of radius R is exposed to an external homogeneous and constant electric field $\vec{E}$, it acquires an electric dipole moment $\vec{d} = \dfrac{\varepsilon - 1}{\varepsilon + 2} R^3 \vec{E}$, where ε is the static dielectric constant. Assume that this relation also applies to slowly time-dependent fields, and thus find the total cross section for Rayleigh scattering.

Problem 3 (40 marks)
A knife edge is formed by a thin conducting sheet that fills the half-plane defined by $x < 0$ and $z = 0$; a plane wave $\vec{E}_{\mathrm{inc}}(\vec{r}, t) = \vec{e}_y \mathrm{Re}\Big(E_0\, \mathrm{e}^{\mathrm{i}kz - \mathrm{i}\omega t}\Big)$ with $k = \dfrac{\omega}{c}$ is incident from the $z < 0$ side. Write $\vec{E}(\vec{r}, t) = \vec{e}_y \mathrm{Re}\Big(E(x, z)\, \mathrm{e}^{\mathrm{i}kz - \mathrm{i}\omega t}\Big)$ for the total electric field (incident plus diffracted).

(a) Use the Huygens approximation to determine $|E(x, z)/E_0|^2$ for $z \gg \lambda = \dfrac{2\pi}{k}$. Your result should involve the Fresnel integral $\mathrm{F}(T)$ of (11.6.29) with argument $T = \sqrt{\dfrac{2}{\lambda z}}\, x$ and correspond to this plot:



The dashed line indicates the geometrical shadow of ray optics.

(b) Use a standard approximation technique (hint: integration by parts) to derive the large-T form of $\mathrm{F}(T)$, and so establish

$$\left|\frac{E(x, z)}{E_0}\right|^2 = \begin{cases} 1 + \dfrac{\sqrt{\lambda z}}{\pi x} \cos\!\Big(\dfrac{\pi x^2}{\lambda z} - \dfrac{3\pi}{4}\Big) & \text{for} \quad x \gg \sqrt{\lambda z}\,, \\ \dfrac{\lambda z}{(2\pi x)^2} & \text{for} \quad -x \gg \sqrt{\lambda z}\,. \end{cases}$$

Use this for a <u>simple</u> estimate of the height of the first maximum outside the geometrical shadow.