The Problem (100 marks)
Consider this ring antenna: a thin wire in the shape of a circle with radius a in the x, y plane, centered at $\vec{r} = 0$, that carries a periodic current $I\cos(\omega t)$. The electric current density is

$$\vec{j}(\vec{r}, t) = I\cos(\omega t)\,\delta(r - a)\,\delta(z)\,\vec{e}_z \times \frac{\vec{r}}{r},$$

and we are interested in $\frac{\mathrm{d}P}{\mathrm{d}\Omega}$, the angular distribution of the radiated power, averaged over one period.

(a) Explain why $a\omega \ll c$ means a "small" antenna? What is $\frac{\mathrm{d}P}{\mathrm{d}\Omega}$ in the small-antenna limit? What is the corresponding total power $P = \int \mathrm{d}\Omega\, \frac{\mathrm{d}P}{\mathrm{d}\Omega}$?

(b) Find $\frac{\mathrm{d}P}{\mathrm{d}\Omega}$ without any assumption about the value of $a\omega/c$.

(c) Sketch the pattern of the angular distribution of the radiation emitted by the ring antenna for $a\omega \ll c$ and $a\omega \gg c$ and describe how the patterns differ. The plot below of the Bessel functions $\mathrm{J}_0(x)$ and $\mathrm{J}_1(x)$ could be helpful.