1 All points on the circle at $z=0$ are the same distance $\sqrt{R^2+z^2}$ away from $\vec{r}=z\vec{e}_z$, where we want to find the retarded potentials. The retardation condition (6.3.3) is then

$$t_{\rm ret}+\frac{1}{c}\sqrt{R^2+z^2}=t\,,$$

and since $\vec{R}(t_{\rm ret})$ and $\vec{V}(t_{\rm ret})$ are both perpendicular to $\vec{r}$, we have — for *this* geometry — the retarded potentials

$$\Phi(\vec{r},t)=\frac{e}{\sqrt{R^2+z^2}}\quad\text{for}\quad\vec{r}=z\vec{e}_z$$

and

$$\vec{A}(\vec{r},t)=\frac{1}{c}\vec{V}(t_{\rm ret})\frac{e}{\sqrt{R^2+z^2}}\quad\text{for}\quad\vec{r}=z\vec{e}_z\,,$$

where

$$\vec{V}(t_{\rm ret})=v[-\vec{e}_x\,\sin\varphi+\vec{e}_y\,\cos\varphi]$$

with

$$\varphi=\frac{vt_{\rm ret}}{R}=\frac{vt}{R}-\frac{v}{c}\sqrt{1+(z/R)^2}\,.$$

---

2 The charge and current densities are

$$\begin{aligned}\rho(\vec{r})&=\frac{e}{4\pi R^2}\delta(r-R)\,,\\ \vec{j}(\vec{r})&=\vec{\omega}\times\vec{r}\rho(\vec{r},t)=\vec{\nabla}\times\left[\frac{e}{4\pi R}\vec{\omega}\eta(R-r)\right].\end{aligned}$$

The electric field is

$$\vec{E}(\vec{r})=\frac{\vec{r}}{r^3}4\pi\int_0^r\mathrm{d}r'\,r'^2\frac{e}{4\pi R^2}\delta(r'-R)=\eta(r-R)e\frac{\vec{r}}{r^3}\,,$$

that is: no electric field inside the sphere, and outside it is the Coulomb field of the net charge.

The magnetic dipole moment is

$$\vec{\mu}=\frac{e}{8\pi Rc}\int(\mathrm{d}\vec{r})\,\underbrace{\vec{r}\times\left[\vec{\nabla}\times\vec{\omega}\eta(R-r)\right]}_{\to 2\vec{\omega}\eta(R-r)\text{ [see (3.6.8)]}}=\frac{e}{4\pi Rc}\vec{\omega}\frac{4\pi R^3}{3}=\frac{eR^2}{3c}\vec{\omega}\,,$$

so that

$$\vec{j}(\vec{r})=\vec{\nabla}\times\left[\frac{3c}{4\pi R^3}\vec{\mu}\eta(R-r)\right].$$

This gives first the vector potential

$$
\begin{aligned}
\vec{A}(\vec{r}) &= \frac{1}{c}\int(\mathrm{d}\vec{r}\,')\,\frac{\vec{j}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} = -\frac{3}{4\pi R^3}\vec{\mu}\times\int(\mathrm{d}\vec{r}\,')\,\frac{1}{|\vec{r}-\vec{r}\,'|}\vec{\nabla}'\eta(R-r') \\
&= -\frac{3}{4\pi R^3}\vec{\mu}\times\vec{\nabla}\int(\mathrm{d}\vec{r}\,')\,\frac{1}{|\vec{r}-\vec{r}\,'|}\eta(R-r') \\
&= \frac{3}{4\pi R^3}\vec{\mu}\times\frac{\vec{r}}{r^3}4\pi\int_0^r \mathrm{d}r'\,r'^2\eta(R-r') \\
&= \frac{\vec{\mu}\times\vec{r}}{R^3r^3}\min\{R^3,r^3\} = \frac{\vec{\mu}\times\vec{r}}{\max\{R^3,r^3\}} \\
&= \eta(R-r)\frac{\vec{\mu}\times\vec{r}}{R^3} + \eta(r-R)\frac{\vec{\mu}\times\vec{r}}{r^3}
\end{aligned}
$$

and then the magnetic field

$$
\vec{B}(\vec{r}) = \vec{\nabla}\times\vec{A}(\vec{r}) = \eta(R-r)\frac{2\vec{\mu}}{R^3} + \eta(r-R)\frac{3\vec{\mu}\cdot\vec{r}\,\vec{r}-r^2\vec{\mu}}{r^5}\,,
$$

that is: a constant magnetic field inside the sphere, and outside it is the field of the magnetic dipole $\vec{\mu}$.

---

3 In (see Exercise 33)

$$
-\frac{\mathrm{d}E}{\mathrm{d}t}\bigg|_{\mathrm{rad}} = \frac{2e^2}{3c^3}\gamma^6\left[\left(\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}\right)^2 - \left(\frac{\vec{v}}{c}\times\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}\right)^2\right]
$$

we insert what applies for motion on a circle with constant speed, that is:

$$
\frac{\mathrm{d}}{\mathrm{d}t}\vec{v} = \vec{\omega}_0\times\vec{v}\,,\quad v=\omega_0R\,,\quad \vec{v}\times\frac{\mathrm{d}}{\mathrm{d}t}\vec{v} = v^2\vec{\omega}_0\,,
$$

and so get

$$
-\frac{\mathrm{d}E}{\mathrm{d}t}\bigg|_{\mathrm{rad}} = \frac{2e^2}{3c^3}\gamma^6\underbrace{\left[(\omega_0v)^2-(v^2\omega_0/c)^2\right]}_{=(\omega_0v)^2/\gamma^2} = \frac{2e^2}{3c^3}\gamma^4(\omega_0v)^2 = \frac{2}{3}\omega_0\frac{e^2}{R}\beta^3\gamma^4\,,
$$

which is the familiar result.