1 As we know, the electric field is

$$\vec{E}(\vec{r}) = \eta(r-R)e\frac{\vec{r}}{r^3}\,,$$

and the magnetic field is

$$\vec{B}(\vec{r}) = \eta(R-r)\frac{2\vec{\mu}}{R^3} + \eta(r-R)\frac{3\vec{\mu}\cdot\vec{r}\,\vec{r} - r^2\vec{\mu}}{r^5}\,,$$

where

$$\vec{\mu} = \frac{eR^2}{3c}\vec{\omega}$$

is the magnetic dipole moment.

(a) We have the energy density

$$U = \frac{1}{8\pi}\eta(R-r)\Big(\frac{2\vec{\mu}}{R^3}\Big)^2 + \frac{1}{8\pi}\eta(r-R)\Big[\Big(e\frac{\vec{r}}{r^3}\Big)^2 + \Big(\frac{3\vec{\mu}\cdot\vec{r}\,\vec{r} - r^2\vec{\mu}}{r^5}\Big)^2\Big]\,,$$

for which the integration over the solid angle gives

$$\int \mathrm{d}\Omega\, U = \eta(R-r)\frac{2\mu^2}{R^6} + \eta(r-R)\Big(\frac{e^2}{2r^4} + \frac{\mu^2}{r^6}\Big)\,.$$

Accordingly, the total energy is

$$\int (\mathrm{d}\vec{r})\, U = \int_0^R \mathrm{d}r\, r^2\,\frac{2\mu^2}{R^6} + \int_R^\infty \mathrm{d}r\, r^2\,\Big(\frac{e^2}{2r^4} + \frac{\mu^2}{r^6}\Big) = \frac{e^2}{2R} + \frac{\mu^2}{R^3}\,.$$

(b) The angular momentum density is

$$\vec{r}\times\frac{1}{4\pi c}\Big[\vec{E}(\vec{r})\times\vec{B}(\vec{r})\Big] = \eta(r-R)\frac{1}{4\pi c}\,\frac{e}{r^6}\,\vec{r}\times(\vec{\mu}\times\vec{r})\,,$$

so that

$$\vec{J} = \frac{e}{c}\int_R^\infty \mathrm{d}r\,\frac{1}{r^4}\,\underbrace{\int\frac{\mathrm{d}\Omega}{4\pi}\,\vec{r}\times(\vec{\mu}\times\vec{r})}_{=\frac{2}{3}r^2\vec{\mu}} = \frac{2e\vec{\mu}}{3cR}\,.$$

(c) The energy current density is

$$\vec{r}\times\vec{G} = \vec{S} = \frac{c}{4\pi}\Big[\vec{E}(\vec{r})\times\vec{B}(\vec{r})\Big] = \eta(r-R)\frac{c}{4\pi}\,\frac{e}{r^6}\,\vec{\mu}\times\vec{r}\,.$$

For $r = R+0$ ("just outside"), this gives

$$\vec{S}\Big|_{r=R+0} = \frac{1}{4\pi}\,\frac{e^2}{3R^4}\,\vec{v}\,,$$

where $\vec{v} = \omega\times\vec{r} = R\omega\times\vec{r}/r$ is the velocity of the charge on the surface.

2 We know, from some exercises, that acceleration by a constant force $F = e|\vec{E}|$ gives a rapidity $\theta(t)$ such that $\sinh(\theta(t)) = Ft/(mc)$ and for the product of $\gamma^3 = \cosh(\theta(t))^3$ and $\dfrac{\mathrm{d}v(t)}{\mathrm{d}t} = c\dfrac{\mathrm{d}}{\mathrm{d}t}\tanh\bigl(\theta(t)\bigr)$ we have

$$\gamma^3 \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{F}{m},$$

so that the rate of radiative energy loss is constant,

$$-\frac{\mathrm{d}E}{\mathrm{d}t}\bigg|_{\mathrm{rad}} = \frac{2e^2F^2}{3m^2c^3} = \frac{2}{3}\frac{e^2}{L}\left(\frac{eV}{mc^2}\right)^2\frac{c}{L},$$

where $V = L|\vec{E}|$ is the voltage drop in each half of the tandem accelerator.

The distance traveled in time T is $c\displaystyle\int_0^T \mathrm{d}t\,\tanh\bigl(\theta(t)\bigr)$. For

$$\tanh(\theta(t)) = \frac{Ft}{\sqrt{(mc)^2 + (Ft)^2}} = \frac{\mathrm{d}}{\mathrm{d}t}\sqrt{(mc/F)^2 + t^2},$$

this gives

$$\frac{2L}{c} = \sqrt{\left(\frac{mc}{F}\right)^2 + T^2} - \frac{mc}{F} \quad \text{or} \quad T = 2\sqrt{\left(\frac{L}{c} + \frac{mc}{F}\right)\frac{L}{c}} = \frac{2L}{c}\sqrt{1 + \frac{mc^2}{eV}}$$

for the duration T of the whole acceleration period. It follows that the total energy radiated is

$$\frac{4}{3}\frac{e^2}{L}\left(\frac{eV}{mc^2}\right)^2\sqrt{1 + \frac{mc^2}{eV}}.$$

---

3 Upon recalling (12.2.6) and remembering that $\cos\theta \simeq 1$ for the relevant angles, the relation of (12.3.8) gives the stated differential cross section. Then, since scattering is almost exclusively in the forward direction, we have $k^2\mathrm{d}\Omega = (\mathrm{d}\vec{k}_\perp)$ and so get

$$\begin{aligned}\sigma &= \int(\mathrm{d}\vec{k}_\perp)\left(\frac{1}{2\pi}\right)^2 \int_{\text{aperture}}(\mathrm{d}\vec{r}_\perp)\,\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{r}_\perp}\int_{\text{aperture}}(\mathrm{d}\vec{r}\,'_\perp)\,\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{r}\,'_\perp} \\ &= \int_{\text{aperture}}(\mathrm{d}\vec{r}_\perp)\int_{\text{aperture}}(\mathrm{d}\vec{r}\,'_\perp)\,\delta(\vec{r}_\perp - \vec{r}\,'_\perp) = \int_{\text{aperture}}(\mathrm{d}\vec{r}_\perp),\end{aligned}$$

so that σ is the area of the aperture.