1. Electromagnetic energy and angular momentum (20=10+6+4 marks)

A spherical shell of radius R with charge e uniformly distributed over its surface rotates about an axis through its center at an angular frequency ω as shown in the figure.

(a) Calculate the total energy contained in the electric and magnetic fields.

(b) What is the total angular momentum contained in the electric and magnetic fields?

(c) In which direction is the electromagnetic energy current density just outside the surface of the sphere?

2. Tandem accelerator (20 marks)

In a symmetric tandem accelerator, one first accelerates H^- ions from rest by a constant electric field of strength E toward a thin foil, which is distance L from the H^- source. When passing through the foil, the H^- ion is stripped of both electrons, and the resulting H^+ ion is then accelerated further by a constant electric field of the same strength E until it hits the target that is distance L behind the foil. Ignore the small mass difference between H^- and H^+ and employ the relativistic version of Larmor's energy-loss formula to determine the total energy that is radiated during the two periods of constant-force acceleration.

3. Diffraction by a large aperture (20 marks)

Proceeding from the familiar approximation for the electric field of the diffracted radiation in the situation of large apertures, show that a single large aperture has a differential cross section for diffraction that is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{k}{2\pi}\right)^2 \left| \int_{\text{aperture}} (\mathrm{d}\vec{r}_\perp)\, \mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{r}_\perp} \right|^2 \quad \text{with } \vec{k} = k\vec{n}.$$

Then argue that

$$\mathrm{d}\Omega = \frac{(\mathrm{d}\vec{k}_\perp)}{k^2}$$

applies here and use this to demonstrate that the total cross section is simply the area of the aperture, irrespective of its shape.