Problem 1 (15 marks)
Point masses $m_1, m_2, \ldots, m_J$ have conservative line-of-sight pair forces among them and are exposed to the external forces $\boldsymbol{F}_j^{(\text{ext})} = m_j \boldsymbol{g}$ where $\boldsymbol{g}$ is the same gravitational acceleration for all masses. At time t_0, the initial conditions give values $\boldsymbol{R}_0$, $\boldsymbol{P}_0$, E_0, and $\boldsymbol{L}_0$ to the center-of-mass position $\boldsymbol{R}$, the total momentum $\boldsymbol{P}_{\text{tot}}$, the total energy E_{tot}, and the total angular momentum $\boldsymbol{L}_{\text{tot}}$. Find $\boldsymbol{R}(t)$, $\boldsymbol{P}_{\text{tot}}(t)$, $E_{\text{tot}}(t)$, and $\boldsymbol{L}_{\text{tot}}(t)$.

Problem 2 (20 marks)
A point mass is scattered elastically by an impenetrable sphere with radius R. Invoke "angle of reflection = angle of incidence" and so determine the relation between the scattering angle θ and the impact parameter b. Then find the differential scattering cross section $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$ and the total cross section $\sigma = \int \mathrm{d}\Omega \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$.

Problem 3 (20 marks)
Under the gravitational pull $\boldsymbol{g} = g\boldsymbol{e}_y$, a point mass is moving along the brachistochrone to get from $(x_1, y_1) = (0, 0)$ to $(x_2, y_2) = (a, 0)$. What is the average speed?

Problem 4 (25=10+8+7 marks)
Point mass m is moving along the horizontal x axis. A spring of natural length a and spring constant k connects the mass to point $(0, a)$ on the y axis.

(a) State the Lagrange function $L(t, x, \dot{x})$ and derive the equation of motion for $x(t)$. Is there a conserved quantity?

(b) For the parameterization $x = a \sinh \vartheta$, state the Lagrange function $L(t, \vartheta, \dot{\vartheta})$, and derive the equation of motion for $\vartheta(t)$.

(c) Which approximate equations of motion apply for $|x| \ll a$ and $|\vartheta| \ll 1$?