Problem 1 (25=7+18 marks)
A point mass m is thrown upwards from $\boldsymbol{r}(t=0)=0$ with initial velocity $\boldsymbol{v}(t=0)=v_0\boldsymbol{e}_z$, $v_0>0$, and then moves under the influence of its weight $m\boldsymbol{g}=-mg\boldsymbol{e}_z$ and the Newtonian frictional force $-m\gamma\boldsymbol{v}$. The point mass is highest above ground at time t_1, and it is back at the initial height $z=0$ at time t_2.

(a) Explain by qualitative arguments, why $t_2>2t_1$, that is: it takes longer to fall down than to fly up.

(b) Find expressions that relate t_1 and t_2 to $\gamma v_0/g$ and then use them to demonstrate that $t_2>2t_1$ is indeed the case. — Hint: $\sinh(\vartheta)>\vartheta$ for $\vartheta>0$.

Problem 2 (45=5+8+12+4+16 marks)
A point mass m is moving along the x axis under the influence of the force associated with the potential energy

$$V(x)=F\big(\sqrt{|x|+a}-\sqrt{a}\,\big)^2$$

with constants $F>0$ and $a>0$.

(a) What are the metrical dimensions of F and a?

(b) Which combination of m, F, and a has the metrical dimension of energy? Which combination has the metrical dimension of time?

(c) Which simple expressions approximate $V(x)$ for $|x|\ll a$ and $|x|\gg a$?

(d) What is the period of small-amplitude oscillations?

(e) What is the energy-dependent period $T(E)$ for motion between two turning points? — Hint: The substitution $x=(y^2+2y)a$ could be helpful.

Problem 3 (30=15+15 marks)
Consider the following two force fields:

$$\text{(i)}\quad \boldsymbol{F}\mathrel{\widehat{=}}\lambda\begin{pmatrix} y^2+yz \\ 2xy-z^2+xz \\ 2yz+xy \end{pmatrix}\quad\text{with}\quad \lambda=\text{constant};$$

$$\text{(ii)}\quad \boldsymbol{F}=\boldsymbol{a}\times(\boldsymbol{r}\times\boldsymbol{a})\quad\text{with}\quad \boldsymbol{a}=\text{constant}.$$

(a) Is force (i) conservative? If yes, find a potential energy for it.

(b) Is force (ii) conservative? If yes, find a potential energy for it.