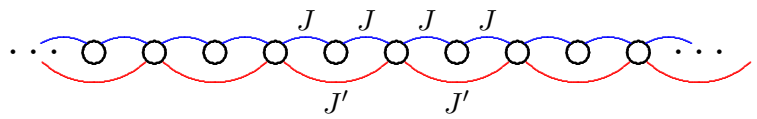
Problem 1 (**25** marks)
For the ideal gas of fermions, and also for the ideal gas of bosons above the critical temperature, we have $\log\bigl(Z(\beta,V,z)\bigr) = \dfrac{V}{\lambda^3}h(z)$ with the appropriate function $h(z)$ of the fugacity z. We know that the adiabatic equation of state $P^3V^5 = \text{constant}$ holds in the dilute-gas limit when $h(z) = z$. What is the adiabatic equation of state for an arbitrary $h(z)$?

Problem 2 (**25** marks)
Consider the standard one-dimensional Ising chain (or ring) with N sites, on-site energy E_0, and next-neighbor interaction strength J. Denote by N_+ and N_- the number of sites with $s_j = +1$ or $s_j = -1$, respectively. Likewise, $N_+^{(\mathrm{nn})}$ is the number of next neighbors with $s_j s_{j+1} = +1$, and $N_-^{(\mathrm{nn})}$ is the number of next neighbors with $s_j s_{j+1} = -1$. What are the expected values of N_+, N_-, $N_+^{(\mathrm{nn})}$, and $N_-^{(\mathrm{nn})}$ in terms of N, βE_0, and βJ?

Problem 3 (**50**=5+5+25+15 marks)
Consider a modified Ising chain (or ring) with N sites, no on-site energy, and next-neighbor interaction strength J. There is also a next-next-neighbor interaction of strength J' between every second site. Symbolically, then, we have this picture:



As usual, we use $K = \beta J$ and $K' = \beta J'$ for convenience.

(a) What is the free energy $F(K, 0, N)$ when $K' = 0$?

(b) What is the free energy $F(0, K', N)$ when $K = 0$?

(c) For $K \neq 0$ and $K' \neq 0$, express the canonical partition function in terms of 2×2 matrices and then find the corresponding free energy $F(K, K', N)$. Verify that you get the expected expressions when $K' = 0$ or $K = 0$.

(d) Determine the heat capacity for $K' = 0$ and find the leading correction when $0 < K' \ll 1$.