Tutorial for Chapter 1

1) Memorize Maxwell's equations in general form and in electrically neutral, non-magnetic materials and be prepared to reproduce them from memory on an exam.

2) Define that $\mathbf{r} = (\hat{x} + 2\hat{y} - 3\hat{z})$ m and $\mathbf{r}_o = (-\hat{x} + 3\hat{y} + 2\hat{z})$ m, (*i*) find the magnitude of $\mathbf{r}$, (*ii*) find the magnitude of $\mathbf{r} - \mathbf{r}_o$, and (*iii*) find the angle between $\mathbf{r}$ and $\mathbf{r}_o$.

3) Define that $\mathbf{F}(\mathbf{r}) = xy\,\hat{x} + (1-yz)\,\hat{y} + xz^2\,\hat{z}$, (*i*) find the divergence of $\mathbf{F}(\mathbf{r})$, and (*ii*) find the carl of $\mathbf{F}(\mathbf{r})$.

4) Define that $f(x, y, z) = 1/(x^2+y^2+z^2)$, find the derivative of $f(x, y, z)$.

5) By applying the same mathematical trick described in Section 1.7, derive the wave equation for the magnetic field $\mathbf{B}$ in vacuum (i.e. $\mathbf{J}_{\text{free}} = 0$ and $\mathbf{P} = 0$).

6) Check that $\mathbf{E}(\mathbf{r}\cdot\hat{\mathbf{u}} - c\,t + \phi)$ satisfies Eq. (1.32), where $\mathbf{E}$ is an arbitrary functional form, $\hat{\mathbf{u}}$ is a unit vector, ϕ is a constant phase, and c is a constant speed.

7) Verify that a summation of waves with different unit vectors $\Sigma_i\,\mathbf{E}_i(\mathbf{r}\cdot\hat{\mathbf{u}}_i - c\,t + \phi_i)$ is also a solution of Eq. (1:32).

8) Suppose that an electric field is given by $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)$, where $\mathbf{k}$ is a constant wave vector perpendicular to $\mathbf{E}_0$, ω is a constant angular frequency, and ϕ is a constant phase.
Show that $\mathbf{B}(\mathbf{r}, t) = \mathbf{k} \times \mathbf{E}_0/\omega \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)$ is consistent with Eq. (1.3).

9) Memorize the wave equations Eq. 1.31 and Eq. 1.32, and be prepared to reproduce them from memory on an exam.

10) Use Microsoft Excel software to produce several snapshots of a wave: $\cos[(x-ct)2\pi/\lambda]$ with $c = 3 \times 10^8$ m/s and $\lambda = 5 \times 10^{-7}$ m, similar to Figure 1.7.

2. i) $|\vec{r}| = \sqrt{1^2+2^2+(-3)^2} = \sqrt{14}$ m

ii) $|\vec{r}-\vec{r}_0| = \sqrt{(1+1)^2+(2-3)^2+(-3-2)^2} = \sqrt{30}$ m

iii) $|\vec{r}_0| = \sqrt{(-1)^2+3^2+2^2} = \sqrt{14}$

$$|\vec{r}|\cdot|\vec{r}_0| = (1)(-1) + (2\times 3) + (-3)\times 2 = -1$$

$$= |\vec{r}||\vec{r}_0| \cos\theta$$

$$\cos\theta = \frac{-1}{\sqrt{14}\sqrt{14}} \qquad \theta = \cos^{-1}\left[\frac{-1}{14}\right]$$

3. i) $\nabla\cdot\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$

$$= y - z + 2xz = y + z(2x-1)$$

ii) $\nabla\times\vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{x}$

$$+ \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{z}$$

$$= (0-(-y)]\hat{x} + [0-z^2]\hat{y} + [0-x]\hat{z}$$

$$= y\hat{x} + (-z^2)\hat{y} + (-x)\hat{z}$$

4. $$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

$$= -\frac{2x\,\hat{x}}{(x^2+y^2+z^2)^2} - \frac{2y\,\hat{y}}{(x^2+y^2+z^2)^2} - \frac{2z\,\hat{z}}{(x^2+y^2+z^2)^2}$$

$$= -\frac{2(x\hat{x}+y\hat{y}+z\hat{z})}{(x^2+y^2+z^2)^2}$$

5. $\because \vec{P}=0, \quad \vec{J}_{free}=0$

$$\therefore \quad \nabla \times \vec{B} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \nabla \times \frac{\vec{B}}{\mu_0} = \varepsilon_0 \frac{\partial}{\partial t}(\nabla \times \vec{E})$$

$$\nabla \times (\nabla \times \vec{B}) = \nabla(\nabla\cdot\vec{B}) - \nabla^2\vec{B} = -\nabla^2\vec{B}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$-\nabla^2\frac{\vec{B}}{\mu_0} = - \quad \frac{\partial}{\partial t}\left(\frac{\partial \vec{B}}{\partial t}\right)$$

$$\nabla^2\vec{B} = \mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}\vec{B}$$

6. $$\nabla^2 \cdot \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \vec{E}$$

$$\vec{E} = \vec{E}(\vec{r} \cdot \hat{u} - ct + \phi)$$

$$\frac{\partial^2}{\partial t^2} \vec{E} = c^2 \vec{E}(\vec{r} \cdot \hat{u} - ct + \phi)$$

$$\nabla^2 \cdot \vec{E} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) \vec{E}$$

$$\frac{\partial^2}{\partial x^2} \vec{E} = \frac{\partial^2}{\partial x^2} \vec{E}(x u_x + y u_y + z u_z - ct + \phi)$$

$$= u_x^2 \vec{E}(x u_x + y u_y + z u_z - ct + \phi)$$

$$\frac{\partial^2}{\partial y^2} \vec{E} = u_y^2 \vec{E}(x u_x + y u_y + z u_z - ct + \phi)$$

$$\frac{\partial^2}{\partial z^2} \vec{E} = u_z^2 \vec{E}(x u_x + y u_y + z u_z - ct + \phi)$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) \vec{E} = (u_x^2 + u_y^2 + u_z^2) \vec{E}$$

$$= \vec{E}$$

$$\vec{E} = \mu_0 \varepsilon_0 c^2 \vec{E} = \vec{E}$$

7. $\nabla^2 \vec{E} = \nabla^2 \sum_i \vec{E_i}$

$= \sum_i \nabla^2 \vec{E_i}$

$= \sum_i \vec{E_i}$

$\mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \sum_i \vec{E_i}$

$= \mu_0 \varepsilon_0 \sum_i \frac{\partial^2}{\partial t^2} \vec{E_i}$

$= \mu_0 \varepsilon_0 \sum_i c^2 \vec{E_i}$

$= \sum_i \vec{E_i}$

Thus $\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \vec{E_i}$

8. $\vec{E} = \vec{E}_0 \cos(\vec{k}\cdot\vec{r} - \omega t + \phi)$

$$= (E_x \hat{x} + E_y \hat{y} + E_z \hat{z}) \cos(\underbrace{k_x x + k_y y + k_z z - \omega t + \phi}_{\theta})$$

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x \cos\theta & E_y \cos\theta & E_z \cos\theta \end{vmatrix}$$

$$= (-E_z \sin\theta\, k_y + E_y \sin\theta\, k_z)\hat{x}$$

$$+ (-E_x \sin\theta\, k_z + E_z \sin\theta\, k_x)\hat{y}$$

$$+ (-E_y \sin\theta\, k_x + E_x \sin\theta\, k_y)\hat{z}$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{k} \times \vec{E}_0/\omega \ \sin\theta\, (-\omega)(-1)$$

$$= \vec{k} \times \vec{E}_0 \sin\theta$$

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ k_x & k_y & k_z \\ E_x & E_y & E_z \end{vmatrix} \sin\theta$$

$$= (k_y E_z - k_z E_y)\hat{x} \sin\theta$$

$$+ (k_z E_x - k_x E_y)\hat{y} \sin\theta$$

$$+ (k_x E_y - k_y E_x)\hat{z} \sin\theta$$