Tutorial for Chapter 2

1. Memorize parameters that characterize a plane wave: frequency, wavelength, wave vector, field amplitudes, and relations among them by (2.2), (2.3), (2.4), (2.5) and (2.6).

2. By ignoring the magnetic field, memorize that a plane wave in complex notation is given by (2.9).

3. Memorize parameters that characterize the material's response to a plane wave: complex index of refraction (2.21), and absorption coefficient (2.62).

4. Memorize the fact that the complex index of refraction is a function of the material and the frequency of light; and memorize (2.37) for dielectrics.

5. Derive the Sellmeier equation

$$n^2 = 1 + \frac{A\lambda_{vac}^2}{\lambda_{vac}^2 - \lambda_{o,vac}^2}$$

from (2.37) for a gas or glass with negligible absorption (i.e. $\gamma \approx 0$, valid far from resonance ω_o), where $\lambda_{o,vac}$ corresponds to frequency ω_o and A is a constant.

6. Many materials (e.g. glass, air) have strong resonances in the ultraviolet. In such materials, do you expect the refractive index for blue light to be greater than that for red light? Make a sketch of n as a function of wavelength for visible light down to the ultraviolet (where $\lambda_{o,vac}$ is located) by using the Sellmeier equation.

7. A complex number is given by $\dfrac{3+i5}{9-i6}$. Find the following:

   (*a*) its complex conjugate;
   (*b*) its real part;
   (*c*) its imaginary part; and
   (*d*) its magnitude and phase.

8. In the dielectric model, take $N = 10^{28}/\text{m}^3$ for the density of bound electrons in an insulator, a resonant frequency $\omega_o = 6 \times 10^{15}$ rad/sec (in the UV), and damping $\gamma = \omega_o/5$ (quite broad). Assume $|\mathbf{E}_0|$ is 10^4 V/m. You don't need to worry about vector directions. For three frequencies $\omega = \omega_o - 2\gamma$, $\omega = \omega_o$, and $\omega = \omega_o + 2\gamma$, find the magnitude and phase of the following (give the phase relative to the phase of $\mathbf{E}_0$). Give correct SI units with each quantity.

   (*a*) The charge displacement amplitude $|\mathbf{r}_{\text{micro}}|$ from Eq. (2.32).
   (*b*) The polarization amplitude $|\mathbf{P}(\omega)|$.
   (*c*) The susceptibility $\chi(\omega)$.

9. Memorize that the electric filed of light can not be directly measured by current technology and the measurable parameter is the intensity of light, which is related to the electric field by (2.61).

10. A short square laser pulse (linearly polarized) with duration $\Delta t = 2.5 \times 10^{-14}$ s and energy $E = 100$ mJ, focused in vacuum to a round spot with radius $r = 5$ μm. Find the following:

   (*a*) its intensity (in W/cm^2);
   (*b*) its peak electric field (in V/Å), HINT: the units of electric field are N/C = V/m; and
   (*c*) its peak magnetic field [in T = kg/(s C)].

5. $$(n + ik)^2 = 1 + \frac{\omega_p^2}{\omega_0^2 - i\omega\gamma - \omega^2}$$

$\because \gamma \simeq 0 \quad \therefore k \simeq 0$

$$n^2 = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2}$$

$$= 1 + \frac{\omega_p^2}{\left(\frac{2\pi c}{\lambda_0}\right)^2 - \left(\frac{2\pi c}{\lambda}\right)^2}$$

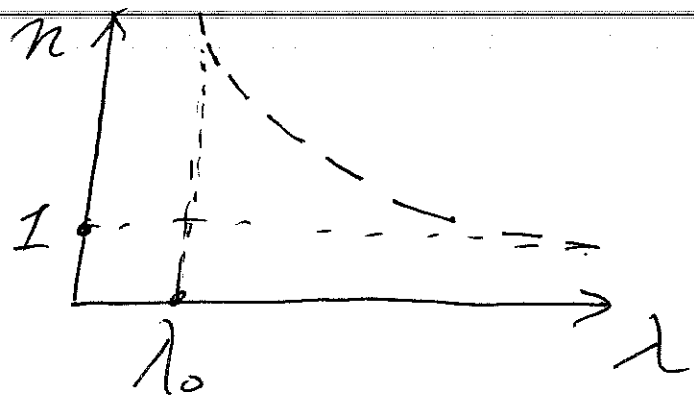
$$= 1 + \frac{\omega_p^2}{4\pi^2 c^2} \frac{\lambda^2 \lambda_0^2}{\lambda^2 - \lambda_0^2}$$

order $$\frac{\omega_p^2 \lambda_0^2}{4\pi^2 c^2} = A$$

$$n^2 = 1 + \frac{A\lambda^2}{\lambda^2 - \lambda_0^2}$$

$$n = \sqrt{1 + \frac{A\lambda_0^2}{\lambda^2 - \lambda_0^2}}$$

6. Yes, expect that n in blue is greater than n in red.



7. (a) $\frac{3-i5}{9+i6}$

(b) $Re\left\{\frac{3+i5}{9-i6}\right\} = Re\left\{\frac{3+i5}{9-i6}\cdot\frac{9+i6}{9+i6}\right\}$

$= Re\left\{\frac{27-5\times6+i(\ldots)}{9^2-(i6)^2}\right\} = Re\left\{\frac{-3+i(\ldots)}{81+36}\right\}$

$= -\frac{3}{117}$

(c) $Im\left\{\frac{3+i5}{9-i6}\right\} = \frac{3\times6+5\times9}{117} = \frac{63}{117}$

(d) $\sqrt{\frac{3+i5}{9-i6} \cdot \frac{3-i5}{9+i6}} = \sqrt{\frac{9+25}{81+36}}$

$= \sqrt{\frac{34}{117}}$

$$\text{Phase} = \tan^{-1}\left(\frac{\text{Im}\{\ \}}{\text{Re}\{\ \}}\right)$$

$$= \tan^{-1}\left\{\frac{\frac{63}{117}}{-\frac{3}{117}}\right\} = \tan^{-1}\left\{-\frac{63}{3}\right\}$$

8. (a) $$|\vec{r}|^2 = \frac{1}{m_e}\frac{q_e E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}}{\omega_0^2 - i\omega\gamma - \omega^2} \cdot \frac{1}{m_e}\frac{q_e E_0 e^{-i(\vec{k}\cdot\vec{r}-\omega t)}}{\omega_0^2 + i\omega\gamma - \omega^2}$$

$$= \frac{q_e^2 E_0^2}{m_e^2}\frac{1}{(\omega_0^2-\omega^2)^2+(\omega\gamma)^2}$$

$$|\vec{r}| = \frac{q_e E_0}{m_e}\frac{1}{\sqrt{(\omega_0^2-\omega^2)^2+(\omega\gamma)^2}}$$

$\omega = \omega_0 - 2\gamma$, $\gamma = \omega_0/5$, $\rightarrow \omega = \omega_0 - 2\omega_0/5 = \frac{3}{5}\omega_0$

$$|\vec{r}| = \frac{q_e E_0}{m_e}\frac{1}{\sqrt{(\omega_0^2 - \frac{3^2}{5^2}\omega_0^2)^2 + (\frac{3}{5}\omega_0\frac{\omega_0}{5})^2}}$$

$$= \frac{q_e E_0}{m_e \omega_0}\frac{1}{\sqrt{(1-\frac{9}{25})^2 + \frac{3^2}{25^2}}} = 0.7\times10^{-16}\,m$$ #

$\omega = \omega_0$, $\gamma = \omega_0/5$

$$|\vec{r}| = \frac{q_e E_0}{m_e}\frac{1}{\sqrt{0^2 + (\omega_0\frac{\omega_0}{5})^2}} = 2.4\times10^{-16}\,m$$ #

$\omega = \omega_0 + 2\gamma$, $\gamma = \omega_0/5 \rightarrow \omega = \omega_0 + \frac{2\omega_0}{5} = \frac{7}{5}\omega_0$

$$|\vec{r}| = \frac{q_e E_0}{m_e}\frac{1}{\sqrt{(\omega_0^2 - \frac{7^2}{5^2}\omega_0^2)^2 + (\frac{7}{5}\omega_0\frac{\omega_0}{5})^2}}$$

$$= 0.48\times10^{-16}\,m$$

(4)

(b) $|\vec{P}| = q_e N |\vec{r}|$

$\omega = \omega_0 - 2\gamma$ $\qquad |\vec{P}| = 1.1 \times 10^{-7}\ C/m^2$

$\omega = \omega_0$ $\qquad |\vec{P}| = 3.8 \times 10^{-7}\ C/m^2$

$\omega = \omega_0 + 2\gamma$ $\qquad |\vec{P}| = 0.77 \times 10^{-7}\ C/m^2$

(c) $$\chi(\omega) = \frac{N q_e^2}{\varepsilon_0 m_e} \frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma}$$

$$= \frac{N q_e^2}{\varepsilon_0 m_e} \frac{(\omega_0^2 - \omega^2) + i\omega\gamma}{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}$$

$\omega = \omega_0 - 2\gamma$, $\gamma = \frac{\omega_0}{5}$ $\rightarrow$ $\omega = \frac{3}{5}\omega_0$

$$\chi(\omega) = \frac{N q_e^2}{\varepsilon_0 m_e} \frac{(\omega_0^2 - \frac{3^2}{5^2}\omega_0^2) + i\frac{\omega_0}{5}\frac{3\omega_0}{5}}{(\omega_0^2 - \frac{3^2}{5^2}\omega_0^2)^2 + (\frac{\omega_0}{5}\frac{3\omega_0}{5})^2}$$

$$= \frac{N q_e^2}{\varepsilon_0 m_e \omega_0^2} \frac{(1 - \frac{9}{25}) + i\frac{3}{25}}{(1 - \frac{9}{25})^2 + (\frac{3}{25})^2}$$

$$= \frac{N q_e^2}{\varepsilon_0 m_e \omega_0^2} \frac{(16 + i3)25}{265}$$

$$= \frac{10^{28} \times (1.6\times10^{-19})^2}{8.85\times10^{-12}\times 9.1\times10^{-31}} \cdot \frac{1}{(6\times10^{15})^2} \frac{25}{265} (16 + i3)$$

$$= 0.08 \times (16 + i3) = 1.3 \times (1 + i\frac{3}{16}) \quad (S.I.)$$

$$\text{Phase} = \tan^{-1}(\frac{3}{16})$$

$\omega = \omega_0$

$$\chi(\omega) = \frac{N q_e^2}{\varepsilon_0 m_e} \frac{i\omega_0 \gamma}{(\omega_0\gamma)^2}$$

$$= \frac{N q_e^2}{\varepsilon_0 m_e} \frac{i}{\omega_0 \frac{\omega_0}{5}} = \frac{5 N q_e^2}{\varepsilon_0 m_e \omega_0^2} i$$

$$= 4.4\, i \quad (S.I.)$$

$$\text{Phase} = \pi/2$$

$$\omega = \omega_0 + 2\gamma, \quad \gamma = \frac{\omega_0}{5} \rightarrow \omega = \frac{7}{5}\omega_0$$

$$\chi(\omega) = \frac{N q_e^2}{\varepsilon_0 m_e} \frac{(\omega_0^2 - \frac{7^2}{5^2}\omega_0^2) + i\frac{7}{5}\omega_0\frac{\omega_0}{5}}{(\omega_0^2 - \frac{7^2}{5^2}\omega_0^2)^2 + (\frac{7}{5}\omega_0\frac{\omega_0}{5})^2}$$

$$= \frac{N q_e^2}{\varepsilon_0 m_e \omega_0^2} \frac{(25-49) + i7}{(25-49)^2 + 7^2} \times 25$$

$$= \frac{4.4}{5} \times 25 \frac{-24 + i7}{625} = \frac{4.4}{5} \frac{\times 25 \times 24}{625} (-1 + i\frac{7}{24})$$

$$= 0.84 \, (-1 + i\frac{7}{24}) \, (S.I.) \quad \text{Phase} = \tan^{-1}(-\frac{7}{24})$$

1. (a) Power $= \frac{\text{Energy}}{\text{Time}} = \frac{100\times10^{-3}\,J}{2.5\times10^{-14}\,s}$

$= 4\times10^{12}\,\frac{J}{s}$

Irradiance $= \frac{4\times10^{12}\;J/s}{\pi\times(5\times10^{-4}\,cm)^2}$

$= \frac{4}{\pi\times25}\times10^{20}\;W/cm^2 = 5\times10^{18}\,\frac{W}{cm^2}$

(b)

$$I = \frac{n\,\varepsilon_0\,c}{2}\,|E_0|^2$$

$$|E_0| = \sqrt{\frac{2I}{n\,\varepsilon_0\,c}}$$

$$= \sqrt{\frac{2\times\frac{4}{\pi\times25}\times10^{20}\,\frac{J}{s\,cm^2}}{1\times8.85\times10^{-12}\,\frac{C^2}{N\,m^2}\times3\times10^{8}\,\frac{m}{s}}}$$

$$= \sqrt{\frac{2\times4\times10^{24}}{\pi\times25\times8.85\times3}}\sqrt{\frac{J\cdot N\cdot m^2}{C^2\;m\;(cm)^2}}$$

$$= \sqrt{\frac{10^{24}}{\pi\times25\times3}}\times10^{2}\,\frac{N}{C}$$

$$= 6.5\times10^{12}\,\frac{V}{m}$$

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(c) $|B_0| = \frac{|E_0|}{c}$

$$= \frac{6.5 \times 10^{12} \frac{V}{m}}{3 \times 10^{8} \frac{m}{s}}$$

$$= 2 \times 10^4 \ \frac{V \cdot A \cdot s}{A \ m^2}$$

$$= 2 \times 10^4 \ \frac{\frac{J}{s} \cdot s}{\frac{C}{s} \ m^2}$$

$$= 2 \times 10^4 \ \frac{kg \ m/s^2 \cdot m}{\frac{C}{s} \ m}$$

$$= 2 \times 10^4 \ \frac{kg}{C \cdot s}$$

(8) $= 2 \times 10^4 \ T$