Tutorial for Chapter 3

1. Memorize the mathematical description of polarized light: Eq. (3.4) and Eq. (3.5). Memorize Jones vectors for various polarization states in Table 3.1.

2. Familiarize yourself with Jones matrices for linear polarizers, Eq. (3.35); wave plates, Eq. (3.44) and Eq. (3.45); reflection or transmission from a surface, Eq. (3.50) and Eq. (3.51); and their effects on Jones vectors.

3. Show that $(A\hat{x} + Be^{i\delta}\hat{y})\bullet(A\hat{x} + Be^{i\delta}\hat{y})^* = 1$, as defined in connection with Eq. (3.5).

4. For the following cases, what is the orientation of the major axis, and what is the ellipticity of the light? Case I: $A = B = 1/\sqrt{2}$; $\delta = 0$ Case II: $A = B = 1/\sqrt{2}$; $\delta = \pi/2$; Case III: $A = B = 1/\sqrt{2}$; $\delta = \pi/4$.

5. (*a*) Suppose that linearly polarized light is oriented at an angle α with respect to the horizontal axis (x-axis) (see Table 3.1). What fraction of the original intensity gets through a vertically oriented polarizer?
(*b*) If the original light is right-circularly polarized, what fraction of the original intensity gets through the same polarizer?

6. Horizontally polarized light ($\alpha = 0$) is sent through two polarizers, the first oriented at $\theta_1 = 45°$ and the second at $\theta_2 = 90°$. What fraction of the original intensity emerges? What is the fraction if the ordering of the polarizers is reversed?

7. (*a*) Suppose that linearly polarized light is oriented at an angle α with respect to the horizontal or x-axis. What fraction of the original intensity emerges from a polarizer oriented with its transmission at angle θ from the x-axis?
(*b*) If the original light is right circularly polarized, what fraction of the original intensity emerges from the same polarizer?

8. What is the minimum thickness (called zero-order thickness) of a quartz plate made to operate as a quarter-wave plate for $\lambda_{vac} = 500$ nm? The indices of refraction are $n_{fast} = 1.54424$ and $n_{slow} = 1.55335$.

9. Describe how to use a linear polarizer and a quarter wave plate to determine whether a circularly polarized wave is right-handed or left-handed.

10. Natural light (un-polarized) travels through a vertically-oriented linear polarizer, a half-wave plate that is oriented at angle θ, to the vertical, and a horizontally-oriented linear polarizer. What is the fraction of the light intensity after transmitting through the three devices?

3. $(A\hat{x} + Be^{i\delta}\hat{y})\cdot(A\hat{x} + Be^{i\delta}\hat{y})^*$

$= A\cdot A + Be^{i\delta}\cdot Be^{-i\delta}$

$= A^2 + B^2 = \dfrac{E_{0x}^2}{E_{0x}^2 + E_{0y}^2} + \dfrac{E_{0y}^2}{E_{0x}^2 + E_{0y}^2}$

$= 1$

4. Case I : $\begin{bmatrix} A \\ Be^{i\delta} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \cos 45^\circ \\ \sin 45^\circ \end{bmatrix}$

ellipticity : linear at 45° to x-axis,
$e = 0$

Case II : $\begin{bmatrix} A \\ Be^{i\delta} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}e^{i\pi/2} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}i \end{bmatrix}$

ellipticity : circular, left-handed,
$e = 1$

Case III : $\begin{bmatrix} A \\ Be^{i\delta} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}e^{i\pi/4} \end{bmatrix}$

ellipticity : elliptical, left-handed.

$$\alpha = \frac{1}{2} \tan^{-1}\left(\frac{2AB\cos\delta}{A^2 - B^2}\right)$$

$$= \frac{1}{2} \tan^{-1}\left(\frac{2AB\cos\delta}{\left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2}\right)$$

$$= \frac{1}{2} \tan^{-1}(\infty) = \frac{1}{2} \cdot \frac{\pi}{2} = \frac{\pi}{4}$$

$$E_{max} = |E_{eff}| \sqrt{A^2\cos^2\alpha + B^2\sin^2\alpha + AB\cos\delta\sin 2\alpha}$$

$$= |E_{eff}| \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2 + \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}}$$

$$= |E_{eff}| \sqrt{\frac{1}{2} + \frac{1}{2\sqrt{2}}}$$

$$E_{min} = |E_{eff}| \sqrt{\frac{1}{2} - \frac{1}{2\sqrt{2}}}$$

$$e = \frac{E_{min}}{E_{max}} = \frac{\sqrt{\frac{1}{2} - \frac{1}{2}\frac{1}{\sqrt{2}}}}{\sqrt{\frac{1}{2} + \frac{1}{2\sqrt{2}}}} = \frac{\sqrt{\sqrt{2} - 1}}{\sqrt{\sqrt{2} + 1}}$$

$$= \sqrt{2} - 1$$

5. (a) Linear $\begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$

Polarizer $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

After the polarizer $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$

$= \sin\alpha$

intensity $= \sin^2\alpha$

(b) Circular $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$

Polarizer $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

After the polarizer $\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -i \end{bmatrix}$

$= \frac{1}{\sqrt{2}}(-i)$

intensity $= \frac{-i}{\sqrt{2}}\frac{i}{\sqrt{2}} = \frac{1}{2}$.

6. (a) Horizantal linear $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

1st polarizer $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

2nd polarizer $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

After 2 polarizers

$$\frac{1}{2}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Intensity $= \left(\frac{1}{2}B\right)^2 = \frac{1}{4}$.

(b) After 2 polarizers

$$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Intensity $= 0$

7. (a) linear $\begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$

polarizer $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$

After polarizer

$$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta\cos\alpha + \cos\theta\sin\theta\sin\alpha \\ \cos\theta\sin\theta\cos\alpha + \sin^2\theta\sin\alpha \end{bmatrix}$$

$$|A|^2+|B|^2 = \left(\cos^2\theta + \sin^2\theta\right)\left(\cos\theta\cos\alpha + \sin\theta\sin\alpha\right)^2$$

Intensity $= (\cos(\theta-\alpha))^2$

(b) right circular $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$

After polarizer

$$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} \cos^2\theta - i\cos\theta\sin\theta \\ \cos\theta\sin\theta - i\sin^2\theta \end{bmatrix}$$

Intensity $= \frac{1}{2}\left[\left(\cos\theta + \cos^2\theta\sin^2\theta\right) + \left(\sin\theta + \cos^2\theta\sin^2\theta\right)\right]$

$= \frac{1}{2}\left[\cos^4\theta + \cos^2\theta\sin^2\theta + \cos^2\theta\sin^2\theta + \sin^4\theta\right]$

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$= \frac{1}{2}$

8. $L|(n_{fast} - n_{slow})| = \frac{\lambda}{4}$

$$L = \frac{500}{4} \cdot \frac{1}{1.55335 - 1.54424} \ nm$$

$$= 13.7 \ \mu m$$

9. Right circular $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$

$\frac{\lambda}{4}$ plate at 45° $\frac{e^{i\pi/4}}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}$

After plate $\frac{e^{i\pi/4}}{\sqrt{2}\sqrt{2}}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -i \end{bmatrix} = \frac{e^{i\pi/4}}{(\sqrt{2})^2}\begin{bmatrix} -1 \\ -2i \end{bmatrix}$

Polarizer (vertical) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

After polarizer $\frac{e^{i\pi/4}}{(\sqrt{2})^2}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -2i \end{bmatrix} = \frac{e^{i\pi/4}}{(\sqrt{2})^2}\begin{bmatrix} 0 \\ -2i \end{bmatrix}$

intensity $= \frac{1}{2^2}(0^2 + |-2i|^2) = \frac{4}{4} = 1$

Left circular $= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$

After $\frac{\lambda}{4}$ plate $\frac{e^{i\pi/4}}{2}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}\begin{bmatrix} 1 \\ i \end{bmatrix} = \frac{e^{i\pi/4}}{2}\begin{bmatrix} 2 \\ 0 \end{bmatrix}$

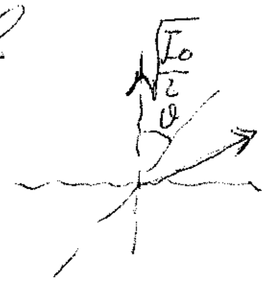
After polarizer $\frac{e^{i\pi/4}}{2}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Intensity $= 0$

Intensity $= I_0$

10. After 1st polarizer $= \frac{I_0}{2}$

E-field $= \sqrt{\frac{I_0}{2}}\sqrt{\frac{2}{\varepsilon_0 n c}}$ in vertical



After $\lambda/2$ plate at θ

E-field rotates by 2θ

After polarizer in horizotal

$$\sqrt{\frac{2}{\varepsilon_0 n c}}\sqrt{\frac{I_0}{2}}\cos(90^\circ - 2\theta)$$

Intensity $= \frac{I_0}{2}\cos^2(90^\circ - 2\theta)$

Fraction $= \frac{1}{2}\cos^2(90^\circ - 2\theta)$

$= \frac{1}{2}\sin^2(2\theta)$