INSTRUCTIONS TO CANDIDATES

1. This examination paper contains five short questions in Part I and three long questions in Part II. It comprises seven printed pages.
2. Answer ALL questions.
3. All answers are to be written on the answer books.
4. This is a CLOSED BOOK examination.
5. The total mark for Part I is 40 and that for Part II is 60.
6. Some useful information are given on Page 2 of this question paper.
Useful Information:

Gravitational constant, \( G \) = \( 6.673 \times 10^{-11} \) N \cdot m^2/kg^2

Acceleration due to gravity, \( g \) = 9.80 m/s\(^2\)

\( I_{CM} \) (for a cylinder) = \( \frac{1}{2} MR^2 \)

\( I_{CM} \) (for a solid sphere) = \( \frac{2}{5} MR^2 \)

\( I_{CM} \) (for a spherical shell) = \( \frac{2}{3} MR^2 \)

\[ \int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha + 1} \quad (\alpha \text{ is real and } \alpha \neq -1) \]

Roots of quadratic equation, \( x \) = \( \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)
1. An egg of mass 0.050 kg is in a stiff box of mass 0.300 kg, surrounded on all sides by padding that constrains the egg in place. A force of \( \vec{F} = (4.61\hat{i} + 2.80\hat{j}) \) N acts on the outside of the box, in addition to the force of gravity (which is oriented in the \(-y\) direction).

(a) Draw the free body diagrams for the egg-box system and for the egg.
(b) What are the acceleration of the egg-box system and the net force on the egg? [8]

2. The axis of the uniform cylinder shown in the figure below is fixed. The cylinder is initially at rest. The block of mass \( M \) is initially moving to the right without friction with speed \( v_1 \). It passes over the cylinder to the dotted position. When it first makes contact with the cylinder, it slips on the cylinder, but friction is large enough so that the slipping ceases before \( M \) loses contact with the cylinder. Find the final speed of the block \( v_2 \) in terms of \( v_1 \), \( M \), radius of the cylinder \( R \) and mass of the cylinder \( m \).
3. A uniform brick of length $L$ is laid on a smooth horizontal surface. Other equal bricks are now piled on as shown, so that the sides from a continuous plane, but the ends are offset at each block from the previous brick by a distance $0.15\ L$. How many bricks can be stacked in this manner before the pile topples over?

![Diagram of bricks stacked in a staircase pattern]

4. A cylinder of density $\rho_0$, length $L$, and cross-section area $A$ floats in a liquid of density $\rho_f$ with its axis perpendicular to the surface. Length $h$ of the cylinder is submerged when the cylinder floats at rest. Show, with clear explanation, that if the cylinder is displaced vertically slightly from its equilibrium position, it oscillates vertically with a period $T$ given by

$$T = 2\pi \sqrt{\frac{h}{g}} .$$

5. The left-hand end of a long, taut string is moved harmonically up and down with amplitude 0.050 m. This motion produces a wave of wavelength $\lambda = 1.50$ m that travels in the $+x$ direction at speed $v = 173$ m/s.

(a) What is the frequency, $f$, of the travelling wave?
(b) What are the maximum transverse velocity and maximum transverse acceleration of a point along the string?
6. Two blocks made of different materials and connected together by a thin cord, slide down a plane ramp inclined at an angle $\theta$ to the horizontal as shown in the diagram below (block 2 is above block 1). The masses of blocks 1 and 2 are $m_1$ and $m_2$, and the coefficients of kinetic friction between the masses and the ramp are $\mu_1$ and $\mu_2$ respectively.

(a) Briefly describe and explain the motion of the two blocks

i. if $\mu_1 < \mu_2$ and

ii. if $\mu_1 > \mu_2$.  [6]

(b) Determine the expressions for the accelerations, $a_1$ and $a_2$, of the blocks and tension $T$ in the cord for both cases above. Simplify your expressions as much as possible. [9]

(c) Explain how the answers to parts (a) and (b) would change (if at all), when the two blocks sliding on the plane are connected by a light rigid rod instead of a cord. [5]
7. (a) A uniform solid ball of radius \( r_0 \) rolls on the inside of a track of radius \( R_0 \) (see figure below). If the ball starts from rest at the vertical edge of the track, what will its speed be when it reaches the lowest point of the track. Assume that the ball always rolls without slipping. \[ 5 \]

(b) For this part of question, take \( r_0 = 1.5 \text{ cm} \) and \( R_0 = 26.0 \text{ cm} \) and the ball starts rolling at height \( R_0 \) above the bottom of the track. It leaves the track after passing through 135° as shown.

i. What will its speed be when it leaves the track?

ii. At what distance \( D \) from the base of the track will the ball hit the ground? (Assume that the track has negligible thickness.) \[ 9 \]

(c) Explain how the results to parts (a) and (b) would change (if at all) when the ball is replaced by another ball of same radius and mass but is hollow, i.e., consider a spherical shell. \[ 6 \]
8. Galileo discovered the four largest moons of Jupiter in 1609. The table below gives the mass of the moons, period of revolution around Jupiter and the mean distance of these moons from Jupiter (measured from the centre of mass of Jupiter to the centre of mass of the moons).

<table>
<thead>
<tr>
<th>Moon</th>
<th>Mass (kg)</th>
<th>Period (Earth Days)</th>
<th>Mean distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Io</td>
<td>$8.9 \times 10^{22}$</td>
<td>1.77</td>
<td>$422 \times 10^3$</td>
</tr>
<tr>
<td>Europa</td>
<td>$4.9 \times 10^{22}$</td>
<td>3.55</td>
<td>$671 \times 10^3$</td>
</tr>
<tr>
<td>Ganymede</td>
<td>$1.5 \times 10^{23}$</td>
<td>7.16</td>
<td>$1070 \times 10^3$</td>
</tr>
<tr>
<td>Callisto</td>
<td>$1.1 \times 10^{23}$</td>
<td>16.7</td>
<td>$1883 \times 10^3$</td>
</tr>
</tbody>
</table>

(a) From the above data, determine the mass of Jupiter. (You may assume circular orbit with radius given by the mean distance and you need not work out the uncertainty.) [6]

(b) The radius of Io is $1.82 \times 10^3$ km. On the surface of Io, what is the value of $g$, i.e., the acceleration due to gravitational pull of Io itself? [2]

(c) A man is on the surface of Io and Jupiter is directly overhead.
   i. What is the magnitude of the force on a ball of mass $m$ due to the gravitation attraction of Jupiter? How is this value compared to that due to Io?
   ii. When the man drops the ball on the surface of Io, would the ball accelerate with $g$ as determined in (b) above or would it be substantially lesser due to the attractive pull of Jupiter. Explain your answer. [6]

(d) What is the energy needed to totally disintegrate the moon Io, i.e., to increase the distance between all individual molecules of Io to infinity? Neglect all other masses around Io and consider only gravitational effects. [6]