PC 4243 - Atomic and Molecular Physics 2

AY06/07 SEM 2

Suggested Solutions

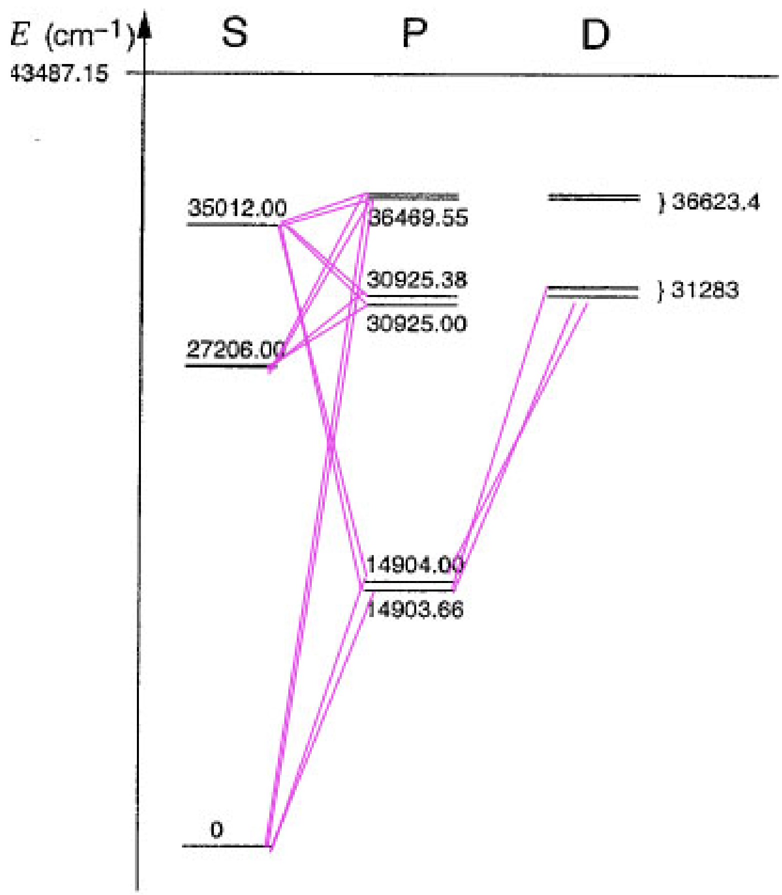
The use of Prof Christian's official solutions as reference is acknowledged.

Q1

a (i)

Li has 3 electrons, and its complete electronic configuration is $1s^2 2s$, with spectroscopic notation $^2S_{1/2}$.

b



The levels in the P and D column appear in pairs because of the spin-orbit interation.

c

The hyperfine interaction leads to a splitting of the ground state.

For ^7Li with $I = 3/2, J = S = 1/2, F = 1, 2$, so that $E_{HFS} = A/2(F(F+1) - J(J+1) - I(I+1))$ gives $E_{HFS} = 3/2A$ for $F = 2$ and $E_{HFS} = -3/4A$ for $F = 1$.

For ^6Li with $I = 1$, $F = 3/2, 1/2$ with corresponding values of $E_{HFS} = A/2A$ and $-A$ respectively.
The physical origin of the hyperfine splitting is due to the contact interaction of s-electrons at the location of the nucleus.

d

The ground state splitting between the 2 levels are given by

$$\begin{aligned}\Delta E_{HFS} &= \frac{A}{2}[(F^+(F^+ + 1) - J(J+1) - I(I+1)) - ((F^+ - 1)(F^+) - J(J+1) - I(I+1))] \\ &= \frac{A}{2}F^+\end{aligned}$$

where F^+ denotes the larger F value. Now $A \propto g_I$ so that $\Delta E_{HFS} \propto g_I F^+$, and we have

$$\begin{aligned}g_I^6 &= \frac{\Delta E_{HFS}^6}{\Delta E_{HFS}^7}\frac{F^{+7}}{F^{+6}} \\ &= 3.26 \cdot \frac{4}{3}\frac{228.2}{803.5} \\ &= 1.23\end{aligned}$$

e

The ionisation energy is proportional to the reduced mass, so that

$$E_1 = E_2\frac{\mu_1}{\mu_2}$$

with

$$\begin{aligned}\mu_1 &= \frac{m_e M_i}{M_i + m_e} \\ &= m_e\left(1 - \frac{m_e}{M_i + m_e}\right) \\ &\approx m_e\left(1 - \frac{m_e}{M_i}\right)\end{aligned}$$

Together,

$$\begin{aligned}
\Delta E &\approx E_1 \frac{m_e}{m_N} \left(\frac{1}{6} - \frac{1}{7} \right) \\
&= E_1 \frac{m_e}{m_N} \\
&= E_1 \frac{1}{1836} \frac{1}{42} \\
&= 0.5 \mathrm{cm}^{-1}
\end{aligned}$$

Q2

a

$$\begin{aligned}\lambda &= \frac{1}{E_e - E_g} \\ &= \frac{1}{62350\text{cm}^{-1} - 44043\text{cm}^{-1}} \\ &= 18307\text{cm}^{-1}\end{aligned}$$

b

${}^3P_2$ means $S = 1, L = 1, J = 2$. 3^S_1 means $S = 1, L = 0, J = 1$. Yes, the transition follows the common selecton rules for dipole transition as $\Delta L = 1$.

c

The homogenous linewidth is due to exponential decay :

$$A(t) = e^{i\omega_0 t} \cdot e^{-t/2\tau}$$

whose fourier transform

$$\begin{aligned}a(\omega) &= \int e^{i(\omega_0 - \omega - 1/2\tau)t}\, dt \\ &= \frac{1}{i(\omega - \omega_0) - 1/2\tau} \\ |a(\omega)|^2 &= \frac{1}{(\omega - \omega_0)^2 + (1/2\tau)^2}\end{aligned}$$

Thus the FWHM $\delta\omega = 2\frac{1}{2\tau} = \frac{1}{\tau}$, giving

$$\delta f_{FWMH} = \frac{1}{2\pi}\frac{1}{\tau} = 19.89\text{MHz}$$

d

The Doppler shift is given by $\Delta f = v\text{f}$, so that

$$v = \Delta f \frac{c}{f} = \Delta f \lambda$$

. Assume a velocity distribution

$$p(v_x) \propto e^{-\frac{1}{2}mv_x^2/kT} = e^{-\frac{\Delta f^2}{2\sigma^2}}$$

with

$$\sigma = \sqrt{\frac{kT}{m}}\frac{1}{\lambda}$$

. To find the Half Width at Half Maximum, we set

$$\begin{aligned} \frac{1}{2} &= e^{-\frac{x^2}{2\sigma^2}} \\ \Rightarrow \quad \ln\frac{1}{2} &= -\frac{x^2}{2\sigma^2} \\ \Rightarrow \quad x &= \sigma\sqrt{2\ln 2} \end{aligned}$$

so that

$$\begin{aligned} \Delta f_{FWHM} &= 2\sqrt{2\ln 2}\sigma \\ &= 2\sqrt{2\ln 2}\sqrt{\frac{kT}{m}}\frac{1}{\lambda} \\ &= 495.99\text{MHz} \end{aligned}$$

e

From $PV = NkT$ we define the number density

$$\rho = \frac{N}{V} = \frac{P}{kT}$$

. For hard core collisions, the collisional cross sectional area $\sigma = D^2\pi$.
From kinetic theory, the mean velocity $v = \sqrt{\frac{2kT}{m}}$.
The mean free path $s = \frac{1}{\sigma\rho} = \frac{kT}{PD^2\pi}$, so that the mean time between collisions

$$\begin{aligned} t &= \frac{s}{v} \\ &= \frac{kT}{P\pi D^2}\sqrt{\frac{m}{2kT}} \\ &= \frac{1}{P\pi D^2}\sqrt{\frac{kTm}{2}} \\ &= 8.6 \times 10^{-5}\text{s}^{-1} \end{aligned}$$

Then

$$\Delta f = \frac{1}{t} = 11.6\text{kHz}.$$

Q3

a

Use σ^+ light and wait for a few spontaneous emissions. The final state is not a dark state with respect to the pumping field, vecause there is an allowed transition $m_i = 1$ to $m_F = 2$.

b

Entropy before :

$$S = -\sum p_i \ln p_i = -3\frac{1}{3}\ln\frac{1}{3} = 1.1$$

The entropy after is 0 since this is a pure state.

c

We need a π transition. Apply a π-polarised light for half a Rabi osciallation.

d

The probability is proportional to $|C.G|^2$. Hence $p_{-1} = 0$, while

$$\frac{p_0}{p_{-1}} = \frac{|-\sqrt{3/10}|^2}{|\sqrt{3/10}|^2} = 1$$

implying that both of these transitions occur with equal probabilities.

Q4

a

The interaction Hamiltonian

$$H(t) = B\langle F=4|\hat{\mu}|F=3\rangle$$

where $\hat{\mu} = \hat{\mu_s} = g_s \hat{s}_z \mu_B/\hbar$. Here,

$$|F=3\rangle = \frac{1}{\sqrt{2}}\left(|m_i = 1/2, m_j = -1/2\rangle + |m_i = -1/2, m_j = 1/2\rangle\right)$$

so that

$$\begin{aligned}
\langle 4|\hat{\mu}_z|3\rangle &= \frac{1}{2}g_s(\langle -1| - \langle +|)\hat{\mu}_z(|-\rangle + |+\rangle) \\
&= \frac{1}{2}g_s(\langle -|s_z|-\rangle - \langle +|s_z|+\rangle) \\
&= \frac{1}{2}g_s\mu_B(-hbar/2 - \hbar/2) \\
&= g_s\frac{\mu_B}{\hbar}\hbar\frac{1}{2} \\
&= \mu_B
\end{aligned}$$

Now

$$\hbar\Omega_R = B_0\mu_B \Rightarrow \Omega_R = \frac{B_0\mu_B}{\hbar}$$

so that

$$\begin{aligned}
B_0 &= \frac{\Omega_R}{\mu_B}\hbar \\
&= \frac{2\pi f_R}{\mu_B}\hbar \\
&= 7.14 \times 10^{-8} T
\end{aligned}$$

b

$$\rho_e = \frac{\Omega_0^2}{\Omega_0^2 + \Delta^2} = \frac{1^2}{1^2 + 2^2} = \frac{1}{5}$$

.

c

No, because $\frac{1}{\sqrt{2}}(\langle -|\pm\rangle + |)$ is an eigenstate of $\hat{s}_x = \frac{1}{2}(\hat{s}_+ + \hat{s}_-)$ so $\langle -|\hat{s}_x|+\rangle = 0$ implying no transfer.