NATIONAL UNIVERSITY OF SINGAPORE

PC4243: Atomic & Molecular Physics II

(Semester I: AY 2009-10)

Time allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This exam paper contains **FOUR** questions and comprises **SIX** printed pages.
2. You have to answer **THREE** questions.
3. This is a CLOSED BOOK examination.
4. You are allowed one double-sided A4 sheet of notes only.
5. Please use only the supplied answer books, and don't mix answers to different problems on the same sheet.
6. There is a table of Clebsch-Gordan coefficients attached.

1: Atomic Structure

(a) Explain the difference between LS and jj coupling in describing the level structure of multi-electron atoms.

(b) The following table gives the electronic configurations and energies (in cm^{-1}) for the first eight excited states of neutral Barium (relative to the $6s^2$ ground state).

Config.	Energy
6s5d	9033.966
	9215.501
	9596.533
	11395.350
6s6p	12266.024
	12636.623
	13514.745
	18060.261

(i) Suggest, with reasons, further quantum numbers to identify these levels.

(ii) Draw an energy level diagram showing the allowed transitions, within the LS coupling regime.

(iii) Explain the appearance of weak lines at $12636.623\,cm^{-1}$ and $8844.760\,cm^{-1}$.

— Please turn over —

2: Laser Cooling

In steady state, the excited state population for a two level atom is given by

$$\rho_{ee} = \frac{1}{2} \frac{I/I_s}{1 + I/I_s + 4\Delta^2/\Gamma^2}$$

where I_s is the saturation intensity, Δ is the laser detuning from the atomic resonance, and Γ is the line-width of the excited state.

(a) Explain how photon scattering results in a force on an atom. Give an expression for the scattering force on a stationary atom.

(b) Give a brief explanation of the operation of a magneto-optic trap (MOT) using diagrams as appropriate. You may restrict your discussion to one spatial dimension for a simple J=0 to J=1 transition. For small velocities and small displacements from the trap center, show that an atom undergoes damped simple harmonic motion.

— Please turn over —

3: Light forces

Consider the $^2S_{1/2}$ to $^2P_{1/2}$ transition in ^{87}Rb. The upper $^2P_{1/2}$ state has a line-width of $2\pi \times 5.7\,$MHz and the wavelength associated with the transition is 795 nm.

(a) Given that the nuclear spin of ^{87}Rb is $I = 3/2$, give a sketch of the level structure you would expect due to the hyperfine interaction and label the levels with the appropriate F quantum numbers.

(b) The hyperfine splitting of the $^2S_{1/2}$ and $^2P_{1/2}$ states are 6.8 GHz and 810 MHz respectively. Calculate the width of absorption lines in a room temperature vapour cell you would expect from these transitions. What features would you be able to resolve?

(c) Calculate the AC Stark shift of all of the F=1 ground-state sub-levels due to a σ^+ polarized laser field with a peak intensity of $90\,\mathrm{W/m^2}$ tuned 600 MHz below the $F = 1$ to $F = 1$ transition.

Note: You may find the following equations useful

$$I_0 = \frac{1}{2}\epsilon_0 c E_0^2, \qquad A_{ij} = \frac{\omega_{ij}^3 \mu_{ij}^2}{3\pi\epsilon_0\hbar c^3}, \qquad \gamma_s = \frac{\Gamma}{2}\frac{2\Omega^2/\Gamma^2}{1 + 2\Omega^2/\Gamma^2 + 4\Delta^2/\Gamma^2}.$$

$$\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F/m}, \quad c = 2.9979 \times 10^8\,\mathrm{m/s}, \quad \hbar = 1.055 \times 10^{-34}\,\mathrm{Js}$$

— Please turn over —

4: Two Level Systems

An electron is subjected to a time varying magnetic field

$$\mathbf{B} = B_{\mathrm{rf}} \cos \omega t \, \hat{\mathbf{x}} + B_{\mathrm{rf}} \sin \omega t \, \hat{\mathbf{y}} + B_0 \hat{\mathbf{z}}$$

The Hamiltonian is given by

$$H = \frac{g_s \mu_B}{\hbar} \mathbf{S} \cdot \mathbf{B}$$

(a) With reference to the Bloch sphere, give a geometric description of the evolution of the state of the electron when $B_{\mathrm{rf}} \ll B_0$. You will find it useful to consider a reference frame in which the magnetic field is stationary.

(b) Show that as the frequency is increased, there is a resonance condition at which the spin flips. Determine the approximate width of this resonance.

(c) Explain why the potential energy of an atom in a magnetic trap is proportional to the magnitude of the magnetic field $|\mathbf{B}|$.

(d) In a magnetic trap, explain how a radio-frequency field may be used to evaporate atoms from the trap.

[MDB]

— End of paper —

32. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

	J J ...
	M M ...
m_1 m_2	Coefficients
m_1 m_2	
. .	

Each table below lists, for each value of M, the coefficients under the column of the total J.

1/2×1/2

M	m_1	m_2	J=1	J=0
+1	+1/2	+1/2	1	
0	+1/2	-1/2	1/2	1/2
0	-1/2	+1/2	1/2	-1/2
-1	-1/2	-1/2	1	

1×1/2

M	m_1	m_2	J=3/2	J=1/2
+3/2	+1	+1/2	1	
+1/2	+1	-1/2	1/3	2/3
+1/2	0	+1/2	2/3	-1/3
-1/2	0	-1/2	2/3	1/3
-1/2	-1	+1/2	1/3	-2/3
-3/2	-1	-1/2	1	

2×1/2

M	m_1	m_2	J=5/2	J=3/2
+5/2	+2	+1/2	1	
+3/2	+2	-1/2	1/5	4/5
+3/2	+1	+1/2	4/5	-1/5
+1/2	+1	-1/2	2/5	3/5
+1/2	0	+1/2	3/5	-2/5
-1/2	0	-1/2	3/5	2/5
-1/2	-1	+1/2	2/5	-3/5
-3/2	-1	-1/2	4/5	1/5
-3/2	-2	+1/2	1/5	-4/5
-5/2	-2	-1/2	1	

3/2×1/2

M	m_1	m_2	J=2	J=1
+2	+3/2	+1/2	1	
+1	+3/2	-1/2	1/4	3/4
+1	+1/2	+1/2	3/4	-1/4
0	+1/2	-1/2	1/2	1/2
0	-1/2	+1/2	1/2	-1/2
-1	-1/2	-1/2	3/4	1/4
-1	-3/2	+1/2	1/4	-3/4
-2	-3/2	-1/2	1	

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

2×1

M	m_1	m_2	J=3	J=2	J=1
+3	+2	+1	1		
+2	+2	0	1/3	2/3	
+2	+1	+1	2/3	-1/3	
+1	+2	-1	1/15	1/3	3/5
+1	+1	0	8/15	1/6	-3/10
+1	0	+1	6/15	-1/2	1/10
0	+1	-1	1/5	1/2	3/10
0	0	0	3/5	0	-2/5
0	-1	+1	1/5	-1/2	3/10
-1	0	-1	6/15	1/2	1/10
-1	-1	0	8/15	-1/6	-3/10
-1	-2	+1	1/15	-1/3	3/5
-2	-1	-1	2/3	1/3	
-2	-2	0	1/3	-2/3	
-3	-2	-1	1		

1×1

M	m_1	m_2	J=2	J=1	J=0
+2	+1	+1	1		
+1	+1	0	1/2	1/2	
+1	0	+1	1/2	-1/2	
0	+1	-1	1/6	1/2	1/3
0	0	0	2/3	0	-1/3
0	-1	+1	1/6	-1/2	1/3
-1	0	-1	1/2	1/2	
-1	-1	0	1/2	-1/2	
-2	-1	-1	1		

3/2×1

M	m_1	m_2	J=5/2	J=3/2	J=1/2
+5/2	+3/2	+1	1		
+3/2	+3/2	0	2/5	3/5	
+3/2	+1/2	+1	3/5	-2/5	
+1/2	+3/2	-1	1/10	2/5	1/2
+1/2	+1/2	0	3/5	1/15	-1/3
+1/2	-1/2	+1	3/10	-8/15	1/6
-1/2	+1/2	-1	3/10	8/15	1/6
-1/2	-1/2	0	3/5	-1/15	-1/3
-1/2	-3/2	+1	1/10	-2/5	1/2
-3/2	-1/2	-1	3/5	2/5	
-3/2	-3/2	0	2/5	-3/5	
-5/2	-3/2	-1	1		

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

2×3/2

M	m_1	m_2	J=7/2	J=5/2	J=3/2	J=1/2
+7/2	+2	+3/2	1			
+5/2	+2	+1/2	3/7	4/7		
+5/2	+1	+3/2	4/7	-3/7		
+3/2	+2	-1/2	1/7	16/35	2/5	
+3/2	+1	+1/2	4/7	1/35	-2/5	
+3/2	0	+3/2	2/7	-18/35	1/5	
+1/2	+2	-3/2	1/35	6/35	2/5	2/5
+1/2	+1	-1/2	12/35	5/14	0	-3/10
+1/2	0	+1/2	18/35	-3/35	-1/5	1/5
+1/2	-1	+3/2	4/35	-27/70	2/5	-1/10
-1/2	+1	-3/2	4/35	27/70	2/5	1/10
-1/2	0	-1/2	18/35	3/35	-1/5	-1/5
-1/2	-1	+1/2	12/35	-5/14	0	3/10
-1/2	-2	+3/2	1/35	-6/35	2/5	-2/5
-3/2	0	-3/2	2/7	18/35	1/5	
-3/2	-1	-1/2	4/7	-1/35	-2/5	
-3/2	-2	+1/2	1/7	-16/35	2/5	
-5/2	-1	-3/2	4/7	3/7		
-5/2	-2	-1/2	3/7	-4/7		
-7/2	-2	-3/2	1			

3/2×3/2

M	m_1	m_2	J=3	J=2	J=1	J=0
+3	+3/2	+3/2	1			
+2	+3/2	+1/2	1/2	1/2		
+2	+1/2	+3/2	1/2	-1/2		
+1	+3/2	-1/2	1/5	1/2	3/10	
+1	+1/2	+1/2	3/5	0	-2/5	
+1	-1/2	+3/2	1/5	-1/2	3/10	
0	+3/2	-3/2	1/20	1/4	9/20	1/4
0	+1/2	-1/2	9/20	1/4	-1/20	-1/4
0	-1/2	+1/2	9/20	-1/4	-1/20	1/4
0	-3/2	+3/2	1/20	-1/4	9/20	-1/4
-1	+1/2	-3/2	1/5	1/2	3/10	
-1	-1/2	-1/2	3/5	0	-2/5	
-1	-3/2	+1/2	1/5	-1/2	3/10	
-2	-1/2	-3/2	1/2	1/2		
-2	-3/2	-1/2	1/2	-1/2		
-3	-3/2	-3/2	1			

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^1_{1,1} = \frac{1+\cos\theta}{2}$$

$$d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$$

$$d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

2×2

M	m_1	m_2	J=4	J=3	J=2	J=1	J=0
+4	+2	+2	1				
+3	+2	+1	1/2	1/2			
+3	+1	+2	1/2	-1/2			
+2	+2	0	3/14	1/2	2/7		
+2	+1	+1	4/7	0	-3/7		
+2	0	+2	3/14	-1/2	2/7		
+1	+2	-1	1/14	3/10	3/7	1/5	
+1	+1	0	3/7	1/5	-1/14	-3/10	
+1	0	+1	3/7	-1/5	-1/14	3/10	
+1	-1	+2	1/14	-3/10	3/7	-1/5	
0	+2	-2	1/70	1/10	2/7	2/5	1/5
0	+1	-1	8/35	2/5	1/14	-1/10	-1/5
0	0	0	18/35	0	-2/7	0	1/5
0	-1	+1	8/35	-2/5	1/14	1/10	-1/5
0	-2	+2	1/70	-1/10	2/7	-2/5	1/5
-1	+1	-2	1/14	3/10	3/7	1/5	
-1	0	-1	3/7	1/5	-1/14	-3/10	
-1	-1	0	3/7	-1/5	-1/14	3/10	
-1	-2	+1	1/14	-3/10	3/7	-1/5	
-2	0	-2	3/14	1/2	2/7		
-2	-1	-1	4/7	0	-3/7		
-2	-2	0	3/14	-1/2	2/7		
-3	-1	-2	1/2	1/2			
-3	-2	-1	1/2	-1/2			
-4	-2	-2	1				

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\,\frac{1+\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\,\frac{1-\cos\theta}{2}\cos\frac{\theta}{2} \qquad d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta \qquad d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta - 1)$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta - 1}{2}\cos\frac{\theta}{2} \qquad d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta \qquad d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta + 1}{2}\sin\frac{\theta}{2} \qquad d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2 \qquad d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta + 1) \qquad d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Figure 32.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The coefficients here have been calculated using computer programs written independently by Cohen and at LBNL.