NATIONAL UNIVERSITY OF SINGAPORE

PC4243: Atomic & Molecular Physics II

(Semester II: AY 2008-09)

Time allowed: 2 hours

<u>INSTRUCTIONS TO CANDIDATES</u>

1. This exam paper contains **FOUR** questions and comprises **SIX** printed pages.

2. You have to answer **THREE** questions.

3. This is a CLOSED BOOK examination

4. Please use only the supplied answer books, and don't mix answers to different problems on the same sheet.

5. There is a table of Clebsch-Gordan coefficients attached.

1: LS-coupling and the Zeeman effect

(a) The interaction of an atom with an external magnetic field is described by the Hamiltonian

$$H_Z = -\frac{\mu_B}{\hbar}(\mathbf{L} + 2\mathbf{S}) \cdot \mathbf{B}$$

Show that, in the LS coupling scheme, the energies of states $|LSJM_J\rangle$ are shifted by an amount

$$E_Z = g_J \mu_B B M_J$$

where

$$g_J = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$$

(b) Neutral Strontium has a ground state configuration [Kr] $5s^2$. The excited state electron configurations 5s5p, and 5s6s have energy levels, measured from the ground state, of 14318, 14504, 14899, 21698, 29039, and 30592 (in units of cm^{-1}). The spectrum is found to have strong lines at 460.8 nm, 679.3 nm, 688.0 nm, and 707.2 nm and a weak line at 689.5 nm.

   (i) Suggest, with reasons, appropriate quantum numbers to identify the given levels.

   (ii) Draw an energy level diagram including the given energy levels and transitions.

   (iii) Comment on the appearance of the weak line at 689.5 nm.

   (iv) A weak magnetic field is applied which splits the observed lines. Give a sketch which illustrates the Zeeman pattern coming from the line at 707.2 nm when observed perpendicular to the direction of the magnetic field. Your sketch need not illustrate relative intensities but should indicate the correct spacings between lines.

2: Absorption

In steady state, the excited state population for a two level atom is given by

$$\rho_{ee} = \frac{1}{2} \frac{I/I_s}{1 + I/I_s + 4\Delta^2/\Gamma^2}$$

where I_s is the saturation intensity, Δ is the laser detuning from the atomic resonance, and Γ is the line-width of the excited state.

(a) Explain what is meant by power broadening and obtain an expression for the power broadened line-width of a two level atom.

(b) By neglecting the motion of the atoms, show that the attenuation of a laser beam propagating through a vapour of two level atoms can be described by

$$\frac{dI}{dz} = -(n_1 - n_2)\sigma(\Delta)I$$

where n_1 and n_2 are the population densities of the ground and excited states respectively and

$$\sigma(\Delta) = \frac{\hbar\omega\Gamma}{2I_s} \frac{1}{1 + 4\Delta^2/\Gamma^2}$$

is the optical scattering cross-section.

(c) Explain how one can modify the equation in (b) to account for the effects of Doppler broadening. Estimate the line-width (FWHM) of a Doppler broadened ^{133}Cs absorption profile in a vapour cell at 300K for a transition at $\lambda = 852$ nm and compare to the natural line-width of $\Gamma = 2\pi \times 5.18$ MHz. Note: $k_B = 1.38 \times 10^{-23}$ J/K, and $m_{Cs} = 133 \times 1.66 \times 10^{-27}$ kg.

(d) Describe one method that can be used to reduce the effects of Doppler broadening in absorption spectroscopy. Your description should include an explanation of how the Doppler broadening effects are reduced.

— Please turn over —

3: Light forces

Consider the $^2S_{1/2}$ to $^2P_{3/2}$ transition in ^{87}Rb. The upper $^2P_{3/2}$ state has a line-width of $2\pi \times 6$ MHz and the wavelength associated with the transition is 780 nm.

(a) Neglecting any hyperfine structure, give a level diagram indicating all the allowed electrical dipole transitions. Determine the Clebsch-Gordan coefficients for all the allowed transitions.

(b) Determine the AC stark shift of the $m_J = +1/2$ ground state level due to a σ^+ polarized laser field with a peak intensity of $90\,$W/m^2 and detuned from resonance by $2\pi \times 300$ MHz.

(c) Estimate the number of photons scattered per second due to the laser field given in (b) and the associated acceleration due to this scattering. Compare with the case in which the laser is resonant with the transition.

(d) **Either**

Give a brief explanation of the operation of a Zeeman slower using diagrams as appropriate. Your explanation should clearly state the purpose of the tapered solenoid. Derive an expression for the appropriate magnetic field profile for a uniform deceleration, a, from an initial velocity v_0.

Or

Give a brief explanation of Sisyphus cooling using diagrams as appropriate. Your explanation should indicate how the final temperature scales with the intensity, I, and detuning, Δ, of the laser beams and what determines the ultimate limit to the cooling process.

**Note:** You may find the following equations useful

$$I_0 = \frac{1}{2}\epsilon_0 c E_0^2, \qquad A_{ij} = \frac{\omega_{ij}^3 \mu_{ij}^2}{3\pi\epsilon_0 \hbar c^3}, \qquad \gamma_s = \frac{\Gamma}{2}\frac{2\Omega^2/\Gamma^2}{1 + 2\Omega^2/\Gamma^2 + 4\Delta^2/\Gamma^2}.$$

$$\epsilon_0 = 8.85 \times 10^{-12}\,\text{F/m}, \quad c = 2.9979 \times 10^8\,\text{m/s}, \quad \hbar = 1.055 \times 10^{-34}\,\text{Js}$$

4: Molecular Physics

(a) Using the fact that the typical average distance between nuclei in a molecule is of the order $a \sim 1\text{Å}$, estimate the order of magnitude of the electronic, vibrational, and rotational energies of a diatomic molecule.

(b) Explain why the rotational degrees of freedom of a molecule can, to a good approximation, be treated as if the molecule were a rigid body.

(c) Consider the boron trifluoride molecule BF_3 which is a trigonal planar molecule as depicted below.
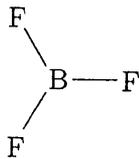
Explain what is meant by orbital hybridization and the role this has in the bonding of the boron trifluoride molecule.

Note: the electronic configurations of boron and fluorine are $1s^2 2s^2 2p^1$ and $1s^2 2s^2 2p^5$ respectively.

(d) Determine the inertia tensor for the boron trifluoride molecule in terms of the B-F bond length, a, and the masses m_B and m_F. Hence determine the quantized energy levels for rotations of the boron trifluoride molecule.

[MDB]

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32. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS,
AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient. *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d_{m,0}^\ell = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d_{m',m}^j = (-1)^{m-m'} d_{m,m'}^j = d_{-m,-m'}^j$$

$$d_{0,0}^1 = \cos\theta \qquad d_{1/2,1/2}^{1/2} = \cos\frac{\theta}{2} \qquad d_{1,1}^1 = \frac{1+\cos\theta}{2}$$

$$d_{1/2,-1/2}^{1/2} = -\sin\frac{\theta}{2} \qquad d_{1,0}^1 = -\frac{\sin\theta}{\sqrt{2}}$$

$$d_{1,-1}^1 = \frac{1-\cos\theta}{2}$$

$$d_{3/2,3/2}^{3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d_{3/2,1/2}^{3/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d_{3/2,-1/2}^{3/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d_{3/2,-3/2}^{3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d_{1/2,1/2}^{3/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d_{1/2,-1/2}^{3/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d_{2,2}^2 = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d_{2,1}^2 = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d_{2,0}^2 = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d_{2,-1}^2 = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d_{2,-2}^2 = \left(\frac{1-\cos\theta}{2}\right)^2$$

$$d_{1,1}^2 = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d_{1,0}^2 = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d_{1,-1}^2 = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

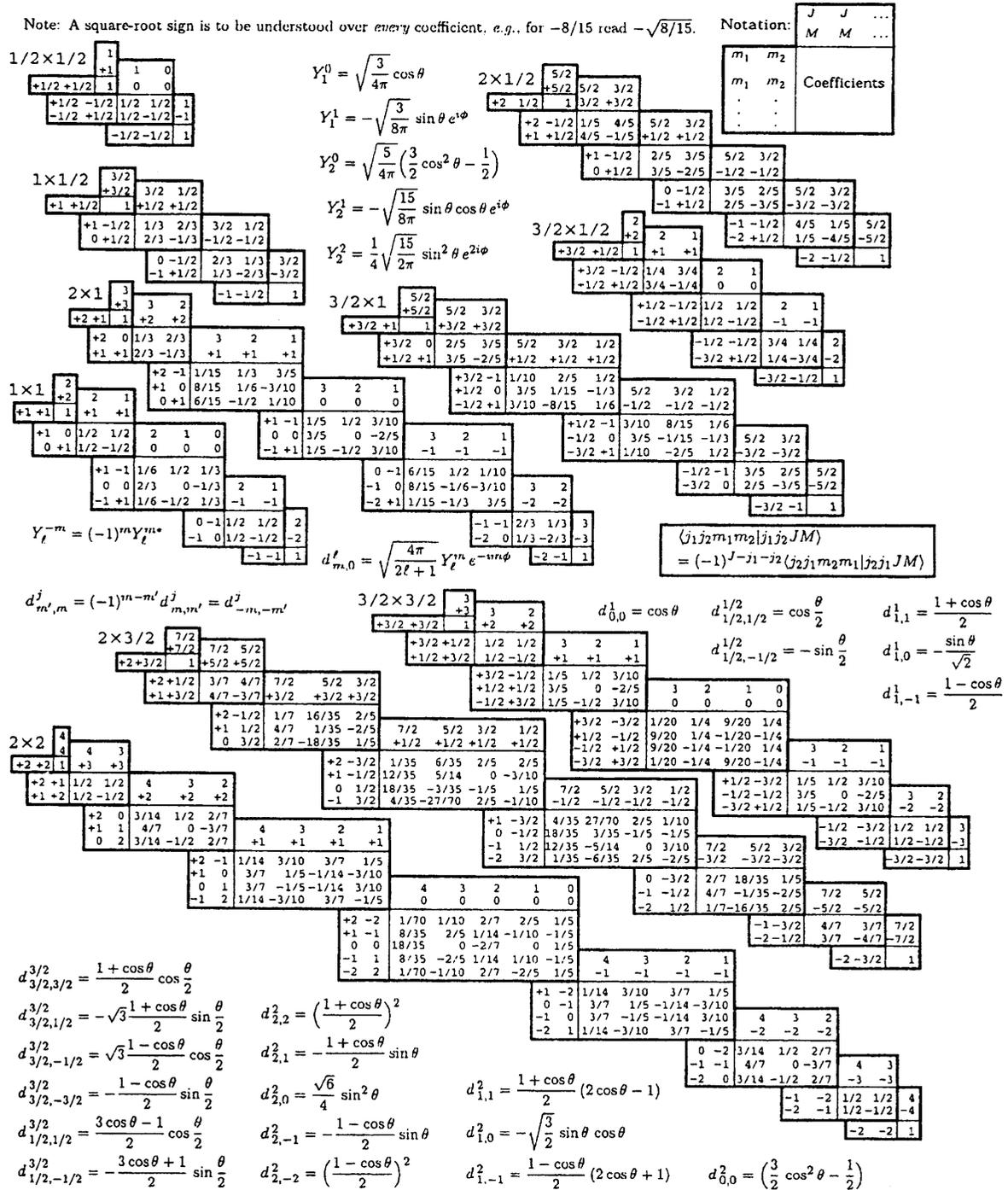$$d_{0,0}^2 = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Figure 32.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The coefficients here have been calculated using computer programs written independently by Cohen and at LBNL.