THE SCIENCE OF MUSIC (HSI2013)
Mid-Term Class Test, Semester 2, 2023/24

This is an open book test. The test is one hour long.

Give your answers to ALL 25 questions on the computer-readable sheet provided, using a soft (2B) pencil to shade the appropriate choice for each question.

1. Which of the following is the most appropriate example of a scientific activity?

   (a) A mechanical engineer performing on a newly invented wind instrument which can be adjusted to play in various types of scales.

   (b) An oboe player rehearsing on an oboe whose mouthpiece is made from a new type of synthetic material.

   (c) A tuba player investigating how tubas made of different metals affect the tone of the tubas.

   (d) An organ technician practising on an electronic organ whose keyboard can give tactile feedback to the player.

   Answer: (c) The mechanical engineer, oboe player and organ technician are all performing essentially musical activities. The tuba player is the only one performing an essentially scientific activity.

2. Which of the following is the most appropriate example of a technological activity?

   (a) A flute player inventing a new type of flute which can also be fitted with a clarinet mouthpiece to be played like a clarinet.

   (b) A clarinet player playing on a new type of clarinet which can electronically produce the lower notes of the bass clarinet.

   (c) An electronic engineer performing on a new type of electronic drum the sound of which can be electronically changed to imitate the tone of a Malay kompang or Chinese lion dance drum.

   (d) A 'cello player rehearsing on a new type of 'cello whose strings can be tuned automatically.

   Answer: (a) The clarinet player, electronic engineer and 'cello player are all performing essentially musical activities. The flute player is the only one performing an essentially technological activity.

3. Which of the following is the most appropriate example of an object undergoing a vibration?

   (a) A single drop of water falling from a tap.

   (b) A piece of ice gradually sliding to a stop on the floor.

   (c) A woman's head moving up and down repeatedly as she falls asleep during a concert.

   (d) A boy jumping up and down just once.

   Answer: (c) The drop of water, piece of ice and boy are each undergoing a single motion and hence not undergoing a vibration. Only the woman's head is undergoing a repeated movement and is therefore undergoing a vibration.

4. A palm tree and a tembusu tree next to it are both swaying repeatedly from left to right during a storm. The tembusu tree undergoes 4 complete cycles in the same time period during which the palm tree undergoes 5 complete cycles. What is the time period during which the tembusu tree would undergo 8 complete cycles if the palm tree undergoes 6 complete cycles in 5 seconds?

(a) 12 seconds.
(b) $\frac{25}{3}$ seconds.
(c) $\frac{25}{6}$ seconds.
(d) None of the above.

Answer: (b) The palm tree undergoes 6 cycles in 5 seconds, and therefore the time it takes to undergo one cycle is given by $\frac{5}{6}$ seconds. The palm tree thus undergoes 5 cycles in $\frac{5}{6}$ seconds times 5 cycles i.e. $\frac{25}{6}$ seconds. The tembusu tree undergoes 4 cycles in the same time period, so the duration of each of the tembusu tree's cycles is equal to $\frac{25}{6}$ seconds divided by 4 i.e. $\frac{25}{24}$ seconds. The tembusu tree will therefore undergo 8 cycles in a duration equal to $\frac{25}{24}$ seconds times 8 i.e. $\frac{25}{3}$ seconds.

5. A necklace is designed by a jewelry designer, and from the jewelry designer's design for the necklace, a jewelry craftsman creates the actual necklace. A young man sees the necklace in a jewelry shop and buys it for his girlfriend who wears the necklace every day. A new song is written by a songwriter, and the song is performed by a famous pop singer for an enthusiastic audience during a pop concert at the National Stadium. Which of the following has the same relationship to the songwriter as the jewelry craftsman has to the jewelry designer?

   (a) The musical score of the song.
   (b) The National Stadium.
   (c) The pop singer.
   (d) The audience.

   Answer: (c). The jewelry craftsman follows the design of the jewelry designer and creates the necklace bought by the young man. The pop singer reads the musical score of the song composed by the songwriter and performs the song for the audience. The pop singer thus has the same relationship to the songwriter as the jewelry craftsman has to the jewelry designer.

6. A junior college choir is beginning its rehearsal for a National Day concert in the college music room by singing the National Day song "We Will Get There". A choir member who is late for the rehearsal then enters the music room. If counterpoint is not to be produced by the choir and the choir member who is late as they sing together, which of the following songs should the choir member sing as she enters the classroom?

   (a) "Home".
   (b) "Stand Up For Singapore".
   (c) "Where I Belong".
   (d) "We Will Get There".

   Answer: (d) Counterpoint is produced only when two or more different melodies are sung at the same time, so the choir member who is late should sing the same melody which is being sung by the rest of the choir, i.e. she should sing "We Will Get There".

7. A police car is passing right in front of you, and the sound of the police car's siren causes a sound level meter you are carrying with you to register a reading of 93 dB. The police car then goes further away from you, and when it is at a traffic light some distance away, the sound level meter registers a reading of 63 dB. How much less is the sound power from the siren when the police car is at the traffic light compared to when it was right in front of you? (Assume that the sound level meter readings are due only to the police car's siren.)

(a) 100 times.
(b) 1,000 times.
(c) 10,000 times.
(d) None of the above.

Answer: (b) The sound power has decreased by 93 dB minus 63 dB i.e. 30 dB when the police car was at the traffic light. Since a decrease of 10 dB means that sound power has decreased by 10 times, if sound power decreases by 30 dB which is 10 dB plus 10 dB plus 10 dB, the sound power would have decreased by 10 times 10 times 10 times i.e. 1,000 times.

8. During a symphony concert a bassoon player plays a note with a frequency of 55 Hz. A violinist then plays a note with a frequency of 1,320 Hz. Which of the following describes the interval between the note played by the bassoon player and the note played by the violinist?

(a) Greater than 1 complete octave but less than 2 complete octaves.
(b) Greater than 2 complete octaves but less than 3 complete octaves.
(c) Greater than 3 complete octaves but less than 4 complete octaves.
(d) Greater than 4 complete octaves but less than 5 complete octaves.

Answer: (d) Since an octave is an interval with a ratio of $\frac{2}{1}$ i.e. 2, if we go up by the interval of an octave this means multiplying the frequency of a musical note by 2. If we start with the bassoon player's note of 55 Hz and multiply this frequency by 2 five times successively, the following frequencies are obtained: 55 Hz, 110 Hz, 220 Hz, 440 Hz, 880 and 1,760 Hz. Therefore the interval from 55 Hz to 1,320 Hz is greater than 4 complete octaves but less than 5 complete octaves.

9. The A4 string of an upright piano is tuned to a frequency of 439 Hz, and all its strings are tuned relative to each other as is normal for a piano. If a certain note on this piano has a frequency of approximately 553.1 Hz, which of the following notes is most likely to be this note? (Take the ratio of an Equal-tempered semitone to be equal to 1.05946 for your calculations.)

(a) Csharp5.
(b) D5.
(c) Dsharp5.
(d) None of the above.

Answer: (a) Since the frequency of the A4 string of the upright piano is 439 Hz, going up from this note by four Equal-tempered semitones, we should multiply 439 Hz by the ratio of an Equal-tempered semitone i.e. 1.05946 four times, which gives a frequency of approximately 553.10 Hz. Hence the note with the frequency of 553.1 Hz is the note which is four semitones above A4 i.e. Csharp5.

10. The score of a melody for solo trumpet starts with a time signature of 33/8. One particular bar of this melody starts with a minim and ends with 2 dotted quavers and 16 semiquavers. Which of the following combinations of notes would fit exactly into the middle of this bar in agreement with the time signature?

(a) 4 dotted crotchets, 3 quavers and 7 semiquavers.
(b) 4 minims and 6 semiquavers.
(c) 3 dotted crotchets, 2 dotted quavers and 11 semiquavers.

(d) 3 crotchets, 5 dotted quavers and 9 semiquavers.

Answer: (d) Since the time signature is 33/8, each bar of the melody should contain the duration equivalent of 33 quavers or 66 semiquavers. The start of the bar has a minim equivalent to 8 semiquavers, and the end of the bar has 2 dotted quavers equivalent to 6 semiquavers and 16 semiquavers, so the bar already has the duration equivalent of 30 semiquavers. The middle of the bar should therefore be filled with the duration equivalent of 36 semiquavers. 3 crotchets are equivalent to 12 semiquavers and with 5 dotted quavers (15 semiquavers)and 9 semiquavers make up a total of 36 semiquavers.

11. You start from the note E6 on the keyboard of a piano and move downwards by an interval of one and a half octaves to arrive at a second note. You then move upwards from this second note, by two-thirds of an octave to arrive at a third note. Moving downwards from this third note by one and five-sixth octaves, you arrive at the fourth and final note. Of the following notes, which one is the correct letter name for the fourth note?

    (a) Aflat3.
    (b) Aflat4.
    (c) A4.
    (d) None of the above.

    Answer: (a) Going down by one and a half octaves or 18 semitones (since an octave consists of 12 semitones) from the starting note E6, we will arrive at the second note Bflat4. Moving up by two-thirds of an octave or 8 semitones brings us to the third note Fsharp5. Finally, moving down from Fsharp5 by one and five-sixth octaves or 22 semitones brings us to the fourth note which is Aflat3.

12. The musical score of a melody for solo trombone consists of a musical staff which starts with a bass clef. A certain bar of this melody has its first note written on the second lowest space of the four spaces of the staff, its second note written on the middle line of the five lines of the staff, its third note written on the highest space of the four spaces of the staff, and its fourth note written on the lowest line of the five lines of the staff. Of the following, which one gives the correct names of these four notes from the first note to the fourth note in the right sequence?

    (a) C3, D3, G3 and A2.
    (b) C3, D3, A3 and G2.
    (c) C3, F3, G3 and G2.
    (d) C3, D3, G3 and G2.

    Answer: (d) The bass clef shows that the second highest line of the staff is the note F3. Therefore the note on the second lowest space of the four spaces of the staff is the note C3, the note on the middle line of the five lines is the note D3, the note on the highest space is the note G3, and the note on the lowest line is the note G2.

13. A guitar's A string is tuned to a frequency of 110 Hz, and all the guitar's strings are tuned in Equal-tempered semitones as is usual for a guitar. A viola's A string is tuned exactly two octaves above the guitar's A string, and all its other strings are tuned relative to each other in Just fifths as is usual for a viola. Of the following, which one is closest to the ratio of the interval between the note from the open G string of the viola and the D4 note of the guitar? Open string means that the notes are not played with a finger on the viola's fingerboard, i.e. they are played with the full length of the respective string vibrating. (Take the ratio of an Equal-tempered semitone to be equal to 1.05946 for your calculations.)

(a) 1.338.
(b) 1.417.
(c) 1.502.
(d) 1.591.

Answer (c) The frequency of the A string of the guitar is 110 Hz i.e. the note is A2. The guitar's A3 thus has a frequency of 220 Hz, and the frequency of its D4 note is five Equal-tempered semitones above its A3 note. This frequency is thus equal to 220 Hz times 1.05946 five times which gives approximately 293.6605 Hz. The A string of the viola has a frequency of 440 Hz which is A4, so the frequency of its G string which is two Just fifths below the A string is equal to 440 Hz divided by $\frac{9}{4}$ or times $\frac{4}{9}$ i.e. approximately 195.5556 Hz which is G3. Therefore the ratio between this G3 note and the guitar's D4 note is approximately equal to 293.6605 Hz divided by 195.5556 Hz i.e. approximately 1.502.

14. You start from a first note and then going up by an interval of an octave and a Pythagorean third, you arrive at a second note. You start again from this second note and then go down by the interval of a Pythagorean fifth and a Pythagorean seventh to arrive at a third note. Finally, you go up from this third note by the interval of an octave and a Just seventh to arrive at a fourth note. Which of the following gives the ratio of the interval between the first note and the fourth note?

(a) $\frac{5}{3}$.
(b) $\frac{10}{3}$.
(c) $\frac{20}{3}$.
(d) None of the above.

Answer: (b) Since the ratio of a Pythagorean third is $\frac{81}{64}$, going up by an octave and a Pythagorean third means multiplying the starting frequency by 2 to go up by an octave and then multiplying by $\frac{81}{64}$. The Pythagorean fifth has a ratio of $\frac{3}{2}$, so going down by a Pythagorean fifth means dividing by $\frac{3}{2}$ i.e. multiplying by $\frac{2}{3}$.The ratio of a Pythagorean seventh is $\frac{243}{128}$, so going down by a Pythagorean seventh means dividing by $\frac{243}{128}$ i.e. multiplying by $\frac{128}{243}$. The ratio of a Just seventh is $\frac{15}{8}$, so going up by an octave and a Just seventh means multiplying by 2 and then by $\frac{15}{8}$. Therefore the frequency of the fourth note as compared to that of the first note is given by 2 times $\frac{81}{64}$ times $\frac{2}{3}$ times $\frac{128}{243}$ times 2 times $\frac{15}{8}$, which gives $\frac{10}{3}$.

15. When you place your finger at a distance of 24 cm from the nearer end of a string, the string vibrates at a frequency of 1,320 Hz. What is the fundamental frequency of the string if its length is 144 cm?

(a) 200 Hz.
(b) 240 Hz.
(c) 440 Hz.
(d) None of the above.

Answer: (d) Since the string is 144 cm long, the string must be vibrating at its 6th harmonic since 24 cm is one-sixth of 144 cm. Therefore the fundamental frequency of the string is given by 1,320 Hz divided by 6 i.e. 220 Hz.

16. A string which is 120 cm long vibrates at a frequency of 1,750 Hz with a certain number of nodes between its two ends. A second string which is 100 cm long vibrates at a frequency of

1,500 Hz with 5 antinodes between its two ends. Find the number of nodes between the two ends of the first string (not counting the nodes at both ends). (Assume that the two strings are identical in all respects except length.)

(a) 6 nodes.
(b) 5 nodes.
(c) 4 nodes.
(d) None of the above.

Answer: (a) Since the second string has 5 antinodes, it is at its 5th harmonic and hence its fundamental frequency is given by 1,500 Hz divided by 5 i.e. 300 Hz. Therefore the fundamental frequency of the first string is equal to 300 Hz times $\frac{100}{120}$ i.e. 250 Hz. Since the first string is vibrating at a frequency of 1,750 Hz, it must be vibrating at its 7th harmonic since 1,750 Hz divided by 250 Hz is equal to 7. Hence the first string must have 7 antinodes and 6 nodes between its two ends (not counting the nodes at both ends).

17. A string labelled C is vibrating at a frequency of 1,440 Hz with 7 nodes between its two ends (not counting the nodes at both ends). A second string labelled D which is 20% longer than C is sliced into 7 pieces of equal length. 3 of these 7 pieces are joined up to make a third string labelled E. Calculate the frequency of vibration of the string E when it vibrates with 3 nodes between its two ends (not counting the nodes at both ends). (Assume that the strings are identical in all respects except length.)

(a) 1,050 Hz.
(b) 1,400 Hz.
(c) 1,440 Hz.
(d) None of the above.

Answer: (b) Since the string C has 7 nodes it must have 8 antinodes and is thus vibrating at its 8th harmonic. Therefore its fundamental frequency is given by 1,440 Hz divided by 8 i.e. 180 Hz. Since the string D is 20% longer than C, its fundamental frequency is equal to 180 Hz times $\frac{1}{1.2}$ i.e. 150 Hz. The string E has a length which is three-sevenths that of the length of D, hence its fundamental frequency is equal to 150 Hz times $\frac{7}{3}$ i.e. 350 Hz. Thus when E vibrates with 3 nodes or 4 antinodes, it will be at its 4th harmonic. Its frequency of vibration will thus be equal to 350 Hz times 4 i.e. 1,400 Hz.

18. A string 80 cm long is vibrating such that the distance between its adjacent nodes is 16 cm. Calculate the wavelength of the note produced by the vibrating string in the air travelling towards a listener, given that the string has a fundamental frequency of 250 Hz. (Assume that the velocity of sound in air is 330 m/s.)

(a) 22 cm.
(b) 33 cm.
(c) 44 cm
(d) None of the above.

Answer: (d) Since the length of each vibrating segment of the string is 16 cm, the number of such segments is given by 80 cm divided by 16 cm i.e. 5. Therefore the string has 5 antinodes and is vibrating at its 5th harmonic with a frequency given by 250 Hz multiplied by 5 i.e. 1,250 Hz. The wavelength is therefore equal to 330 m/s divided by 1,250 Hz i.e. 0.264 m or 26.4 cm.

19. A viola player plays a musical note which produces a sound wave travelling towards a listener with a wavelength of exactly 1.2 metres. Which of the following musical notes is most likely to be the note played by the viola player? (Assume that the velocity of sound is 330 metres per second. Assume also that the notes played by the viola are part of the Just scale and that the A4 note on the viola has a frequency of 440 Hz.)

    (a) C4.
    (b) Csharp4.
    (c) D4.
    (d) None of the above.

    Answer: (b) Since the velocity of sound is 330 metres per second, a musical note which has a wavelength of 1.2 metres has a frequency equal to 330 metres per second divided by 1.2 metres i.e. 275 Hz. Going down from the A4 note or 440 Hz on the viola by an octave, we get the note A3 with a frequency of 220 Hz. Going up from 220 Hz by a Just third by multiplying 220 Hz by $\frac{5}{4}$, we obtain 275 Hz. This is the note a Just third above A3 i.e. Csharp4, so the note played is the note Csharp4.

20. A common pentatonic scale has five notes with a succession of intervals as follows: tone, tone, 3 semitones, tone and 3 semitones. Of the following sequences of notes, which one follows this scale correctly?

    (a) F4, G4, B4, C5, D5, and F5.
    (b) F4, G4, A4, C5, D5 and F5.
    (c) F4, Gsharp4, A4, C5, D5 and F5.
    (d) F4, G4, A4, Csharp5, Dsharp5 and F5.

    Answer: (b) The sequence of notes F4, G4, A4, C5, D5, and F5 gives the correct succession of intervals i.e. tone, tone, 3 semitones, tone and 3 semitones.

21. A vibrating string produces a musical note which has a spectrum showing all its harmonics are present, odd and even, up to its 20th harmonic. A closed pipe produces a note which has a spectrum showing its fundamental frequency and all its harmonics up to its 19th harmonic. The frequency of the 8th line from the left in the spectrum of the closed pipe's note is equal to the frequency of the 9th line from the left in the spectrum of the note from the string. If the string's note has a fundamental frequency of 180 Hz, what is the frequency of the 10th line from the left in the spectrum of the closed pipe's note? (Assume that the frequencies in each spectrum increase from left to right.)

    (a) 1,836 Hz.
    (b) 2,025 Hz.
    (c) 2,052 Hz.
    (d) None of the above.

    Answer: (c) Since the 9th line from the left in the spectrum of the string's note is its 9th harmonic, its frequency is 180 Hz times 9 i.e. 1,620 Hz. The 8th line from the left in the spectrum of the closed pipe's note is its 15th harmonic, so its fundamental frequency is given by 1,620 Hz divided by 15 i.e. 108 Hz. The 10th line from the left in the spectrum of the closed pipe's note is its 19th harmonic whose frequency is given by 108 Hz times 19 i.e. 2,052 Hz.

22. The spectrum of a square wave shows all its harmonics up to the 19th harmonic. A note produced by a newly invented string instrument has a spectrum which shows all its harmonics (even and odd) present up to its 18th harmonic. The frequency of the 5th line from the left in the spectrum of the string instrument's note is the same as the frequency of the 6th line from the left in the spectrum of the square wave. What is the frequency of the 13th line from the left in the spectrum of the string instrument's note if the fundamental frequency of the square wave is 160 Hz? (Assume that the frequencies in each spectrum increase from left to right.)

    (a) 2,340 Hz.
    (b) 2,496 Hz.
    (c) 5,408 Hz.
    (d) None of the above.

    Answer: (d) The square wave has a fundamental frequency of 160 Hz so the frequency of the 6th line in its spectrum which is its 11th harmonic is equal to 160 Hz times 11 i.e. 1,760 Hz. Since the 5th line from the left in the spectrum of the string instrument's note is its 5th harmonic, the fundamental frequency of the note must be given by 1,760 Hz divided by 5 i.e. 352 Hz. The 13th line from the left in the spectrum of the string instrument's note must be its 13th harmonic, so its frequency is equal to 352 Hz times 13 i.e. 4,576 Hz.

23. An open pipe labelled P is vibrating with the same frequency as a closed pipe labelled Q. P is vibrating with 4 nodes and Q is vibrating with 5 nodes between both their two ends (not counting the node at one end for Q). Q is then cut into six short open pipes and one short closed pipe all of equal lengths. Four of these short open pipes are joined up with the short closed pipe to make a closed pipe labelled R. If the fundamental frequency of P is 275 Hz, calculate the frequency of R when it vibrates with 7 nodes between its two ends (not counting the node at one end).

    (a) 2,100 Hz.
    (b) 2,156 Hz.
    (c) 2,380 Hz.
    (d) None of the above.

    Answer: (a) The open pipe P has 4 nodes so it is at its 4th harmonic and its frequency is thus equal to 275 Hz times 4 i.e. 1,100 Hz. Since the closed pipe Q has 5 nodes, it is at its 11th harmonic, and its fundamental frequency is given by 1,100 Hz divided by 11 i.e. 100 Hz. R is five-sevenths the length of the closed pipe Q, so its fundamental frequency is equal to 100 Hz times $\frac{7}{5}$ i.e. 140 Hz. If R vibrates with 7 nodes, it will be at its 15th harmonic, and its frequency will be given by 140 Hz times 15 i.e. 2,100 Hz.

24. An open pipe which is labelled K is cut into 9 short open pipes all of the equal lengths. 5 of these short open pipes are joined up and one end closed up to make a closed pipe labelled L. The other 4 short open pipes are joined up to make an open pipe labelled M. What is the ratio of the interval between the frequency of L when it vibrates with 3 nodes between its two ends (not counting the node at one end), and the frequency of M when it vibrates with 5 nodes between its two ends?

    (a) $\frac{25}{14}$.
    (b) $\frac{25}{7}$.
    (c) $\frac{5}{3}$.

(d) None of the above.

Answer: (a) First we let the frequency of the open pipe K be p Hz. An open pipe which is five-ninths the length of K has a fundamental frequency which is given by p Hz times $\frac{9}{5}$ i.e. $\frac{9p}{5}$ Hz. The closed pipe L has a fundamental frequency which is half of this i.e. $\frac{9p}{10}$ Hz. When L vibrates with 3 nodes it is at its 7th harmonic and its frequency is given by $\frac{9p}{10}$ times 7 i.e. $\frac{63p}{10}$ Hz. Since the open pipe M is four-ninths the length of K, its fundamental frequency is given by p Hz times $\frac{9}{4}$ i.e. $\frac{9p}{4}$ Hz. When M vibrates with 5 nodes it will be at its 5th harmonic and its frequency is then equal to $\frac{9p}{4}$ Hz times 5 i.e. $\frac{45p}{4}$ Hz. The ratio of the interval between $\frac{45p}{4}$ Hz and $\frac{63p}{10}$ Hz is equal to $\frac{45p}{4}$ Hz divided by $\frac{63p}{10}$ Hz i.e. $\frac{450}{252}$ which can be simplified to $\frac{25}{14}$.

25. Arrange the following harmonics in order of increasing frequency:
(i) The eighth harmonic frequency of an open pipe W of length p cm.
(ii) The fourth harmonic frequency of an open pipe X of length $\frac{6p}{11}$ cm.
(iii) The seventh harmonic frequency of a closed pipe Y of length $\frac{4p}{9}$ cm.
(iv) The ninth harmonic frequency of a closed pipe Z of length $\frac{7p}{12}$ cm.

(a) (iv), (ii), (i), (iii).

(b) (iii), (i), (iv), (ii).

(c) (ii), (iv), (iii), (i).

(d) None of the above.

Answer: (c) Let f Hz be the fundamental frequency of the open pipe W of length p cm. Therefore the eighth harmonic frequency of W is equal to $8f$ Hz. The open pipe X of length $\frac{6p}{11}$ cm has a fundamental frequency given by f Hz times $\frac{11p}{6p}$ i.e. $\frac{11f}{6}$ Hz. The frequency of its fourth harmonic is given by $\frac{11f}{6}$ Hz times 4 i.e. $\frac{22f}{3}$ Hz. Since an open pipe with a length of $\frac{4p}{9}$ cm will have a fundamental frequency equal to f Hz times $\frac{9p}{4p}$ i.e. $\frac{9f}{4}$ Hz, the closed pipe Y which has the same length of $\frac{4p}{9}$ cm will have a fundamental frequency half of $\frac{9f}{4}$ Hz i.e. $\frac{9f}{8}$ Hz. Therefore the seventh harmonic frequency of X will be equal to $\frac{9f}{8}$ Hz times 7 i.e. $\frac{63f}{8}$ Hz. The closed pipe Z whose length is $\frac{7p}{12}$ cm has a fundamental frequency equal to $\frac{9f}{8}$ Hz times $\frac{4p}{9}$ divided by $\frac{7p}{12}$ i.e. $\frac{6f}{7}$ Hz. The frequency of the ninth harmonic of Z is equal to $\frac{6f}{7}$ Hz times 9 i.e. $\frac{54f}{7}$ Hz. Hence the harmonics in order of increasing frequency are (ii), (iv), (iii) and (i).

END OF TEST PAPER