Answers to Tutorial No 3, Semester 2, 2023/24

1. A string which has 7 antinodes between its two ends is vibrating with a frequency of 1,120 Hz. A second string which has 5 nodes between its two ends (not counting the nodes at either end) is vibrating at a frequency of 1,680 Hz and is 80 cm long. What is the length of the first string which is vibrating with 7 antinodes? A third string which is 70 cm long is vibrating at a frequency of 1,600 Hz. How many nodes would this third string have between its two ends (not counting the nodes at either end)? (Assume that the three strings are similar in all respects except length.)

   Answer: The first string has 7 antinodes so it must be at its 7th harmonic. Therefore its fundamental frequency is given by 1,120 Hz divided by 7 i.e. 160 Hz. Since the second string is vibrating with 5 nodes it must have 6 antinodes and is vibrating at its 6th harmonic. Therefore its fundamental frequency is given by 1,680 Hz divided by 6 i.e. 280 Hz. The length of the first string is thus equal to 80 cm times $\frac{280}{160}$ i.e. 140 cm. Since the third string is 70 cm long, its fundamental frequency is equal to 160 Hz times $\frac{140}{70}$ i.e. 320 Hz. As it is vibrating at a frequency of 1,600 Hz, it must be vibrating at its 5th harmonic as 1,600 Hz divided by 320 Hz is equal to 5. Hence the third string must have 5 antinodes and 4 nodes between its two ends (not counting the nodes at either end).

2. A canoe is travelling along the surface of the sea in the same direction and the same speed as the water waves on the surface of the sea, and it can be observed that the total length of the canoe is exactly equal to 4 complete wavelengths of the water waves. If the waves are moving with a speed of 1.2 metres per second and have a frequency of 1 Hz, calculate the length of the canoe. The frequency of the waves then increases to 1.2 Hz and the speed of the waves increases to 1.5 metres per second. When this happens, what would be the number of wavelengths of the waves which would exactly equal the length of the canoe?
   Answer: The sea waves have a wavelength equal to 1.2 metres per second divided by 1 Hz i.e. 1.2 metres. The length of the canoe is thus equal to 1.2 metres times 4 i.e. 4.8 metres. When the frequency of the waves increases to 1.2 Hz and the speed of the waves increases to 1.5 metres per second, the wavelength of the waves is then equal to 1.5 metres per second divided by 1.2 Hz i.e. 1.25 metres. The number of wavelengths which would exactly equal the length of the canoe is hence given by 4.8 metres divided by 1.25 metres i.e. 3.84 wavelengths.

3. A string vibrating with 6 antinodes between its two ends has a fundamental frequency of 280 Hz. Its frequency is the same as that of a closed pipe of length k cm which is vibrating with 3 nodes between its two ends (not counting the node at one end).

Calculate the fundamental frequency of the closed pipe. When the closed pipe vibrates with 7 nodes between its two ends (not counting the node at one end), its frequency is the same as that of an open pipe vibrating with 8 antinodes between its two ends (not counting the antinodes at both ends). What is the length of the open pipe?

Answer: The string is vibrating with 6 antinodes so it is at its 6th harmonic. Hence its frequency of vibration is equal to 280 Hz times 6 i.e. 1,680 Hz. Since the closed pipe has 3 nodes it must be at its 7th harmonic and thus its fundamental frequency is given by 1,680 Hz divided by 7 i.e. 240 Hz. When the closed pipe has 7 nodes, it will be at its 15th harmonic. Therefore its frequency of vibration will be equal to 240 Hz times 15 i.e. 3,600 Hz. The open pipe has 8 antinodes, so it will have 9 nodes and will be at its 9th harmonic, and hence its fundamental frequency is given by 3,600 Hz divided by 9 i.e. 400 Hz. An open pipe which has the same length k cm as the closed pipe would have a fundamental frequency double that of the closed pipe i.e. 480 Hz. Therefore the open pipe which has a fundamental frequency of 400 Hz must have a length given by k cm times $\frac{480}{400}$ i.e. $\frac{6k}{5}$ cm.

4. A string which is 34 cm long is vibrating with 4 nodes (not counting the nodes at both ends). The note produced by the string combines with a note from a closed pipe resulting in beats of 15 Hz. The closed pipe has a fundamental frequency of 150 Hz and is

vibrating with 5 nodes between its two ends (not counting the node at one end). The string is then slightly shortened, and the beat frequency decreases (without passing through 0 Hz). What is the fundamental frequency of the string? The string is then shortened from 34 cm to 32.7 cm, and assuming that the beats are still produced by the same harmonics of the string and the closed pipe as before, what is the new beat frequency? If the length of the closed pipe is decreased to 80% of its original length, calculate what the beat frequency would then be, assuming that the string is still 32.7 cm long.

Answer: Since the closed pipe has 5 nodes, it is at its 11th harmonic, and its frequency is thus equal to 150 Hz times 11 i.e. 1,650 Hz. On shortening the string slightly its frequency increases, so if the beat frequency decreases, the frequency of the string must have been lower than that of the closed pipe. Since the beat frequency is 15 Hz, the frequency of the string is equal to 1,650 Hz minus 15 Hz i.e. 1,635 Hz. Since the string has 4 nodes and 5 antinodes it must be at its 5th harmonic and its fundamental frequency is given by 1,635 Hz divided by 5 i.e. 327 Hz. The shortened string has a length of 32.7 cm, so its fundamental frequency would be equal to 327 Hz times $\frac{34}{32.7}$ i.e. 340 Hz. Since its 5th harmonic would then be 340 Hz times 5 i.e. 1,700 Hz, the beat frequency would then change to 1,700 Hz minus 1,650 Hz i.e. 50 Hz. When the length of the closed pipe is decreased to 80% of its original length, its fundamental frequency would change to 150 Hz times $\frac{1}{0.8}$ i.e. 187.5 Hz. Its 11th harmonic would then be equal to 187.5 Hz times 11 i.e. 2,062.5 Hz, and the beat frequency would change to 2,062.5 Hz minus 1,700 Hz i.e. 362.5 Hz.

5. Using an electronic tuner which is producing a musical note with a frequency of 440 Hz to help her, a violinist is tuning her violin's A string. She can hear beats of 5 Hz when the note from the A string combines with the note from the tuner. The violinist then gradually loosens the A string of the violin and the beat frequency gradually decreases (without passing through 0 Hz) to 4 Hz. Calculate the frequency of the note produced by the violin's A string when the beat frequency was equal to 5 Hz. To make the frequency of the A string come as close as possible to 440 Hz, what should the violinist do? If the beat frequency had increased to 6 Hz instead of decreasing when the string was loosened, what would the A string's frequency have been when the beat frequency was 5 Hz?
   Answer: The beat frequency was 5 Hz so the frequency of the A string's note was either 440 Hz minus 5 Hz i.e. 435 Hz, or 440 Hz plus 5 Hz i.e. 445 Hz. On loosening the violin's A string, its frequency would have decreased, so since the beat frequency then decreased to 4 Hz, this meant that the frequency of the A string's note must have moved closer to 440 Hz, so the A string's frequency must have been higher than 440 Hz when the beat frequency was 5 Hz i.e. the A

string's frequency must have been equal to 445 Hz. To bring the frequency of the A string closer to 440 Hz, the violinist should decrease its frequency further by continuing to loosen the A string so that the beat frequency becomes less. When the beat frequency is at zero, the frequency of the A string must then be equal to 440 Hz. If the beat frequency had increased to 6 Hz on loosening the A string, the frequency of the 'cello's note must have been lower than 440 Hz. Hence it must have been at 435 Hz when the beat frequency was 5 Hz.

Scientific Inquiry discussion points

The Equal-tempered scale is obtained by dividing an octave into twelve equal steps. This gives us a scale with twelve notes, which has become the basis of most of Western music, whether classical, popular, folk, rock or any other genre of music. The democratic equality of these twelve notes enables music to modulate into any of the twelve available keys with ease, using just twelve notes in one octave. This greatly simplifies the design of musical instruments and how they are played. But this is not a "perfect" system in mathematical terms, as the important interval of the fifth is not exactly 3/2 as in the Just and Pythagorean scales, but deviates by ever so slightly an amount which is not apparent to most listeners. Are there other examples in science and technology where imperfections are an important part of the basis of an actual working system?

While we may consider mathematical and physical per-

fection to be most desirable, in the real world most things and processes deviate from mathematical perfection. One crucial example is in the DNA of our genetic code. The reproduction of DNA as it replicates in the multiplication of living cells is not perfect, in that errors may occur in the replication due to natural events such as the alteration of the DNA code by natural radiation or cosmic rays. This may seem undesirable and it does lead to undesirable effects sometimes, but this same process makes evolution possible, as the errors in replication allow changes in the make-up of livings things, which may then make an organism less or more suited to the changing environment. Hence the progress brought about by evolution depends on these imperfections in replication.