Answers to Tutorial No 4, Semester 2, 2023/24

1. A closed pipe which has a fundamental frequency k Hz is labelled P, and is sliced into seven pieces of equal length. This creates six short open pipes, labelled P1 to P6, and a short closed pipe P7. Three of the short open pipes P4, P5 and P6 are joined up with P7 to make a closed pipe labelled Q, and the remaining short open pipes P1 to P3 are joined up to make an open pipe labelled R. What are the fundamental frequencies of the pipes P1, P7, Q and R? Calculate the ratio of the interval between the frequency of Q when it vibrates with 4 nodes between its two ends (not counting the node at one end), and the frequency of R when it vibrates with 6 nodes between its two ends.

   Answer: The short closed pipe P7 is one-seventh the length of P, so its fundamental frequency is equal to $7k$ Hz. The short open pipe of the same length will thus have double the fundamental frequency of P7 i.e. $14k$ Hz. Since the closed pipe Q is four times the length of P7, its fundamental frequency will be equal to $\frac{7k}{4}$ Hz. When Q has 4 nodes, it will be at its 9th harmonic and its frequency will thus be given by $\frac{7k}{4}$ Hz times 9 i.e. $\frac{63k}{4}$ Hz. A closed pipe which has a length three times the length of P7 will have a fundamental frequency of $\frac{7k}{3}$ Hz, so R which is an open pipe of the same length will have a fundamental frequency double this i.e. $\frac{14k}{3}$ Hz. When R has 6 nodes it will be at its 6th harmonic and its frequency will then be equal to $\frac{14k}{3}$ Hz times 6 i.e. $\frac{84k}{3}$ or $28k$ Hz. The ratio of the interval between $\frac{84k}{3}$ Hz and $\frac{63k}{4}$ Hz is given by $\frac{84k}{3}$ Hz divided by $\frac{63k}{4}$ Hz i.e. $\frac{28.4k}{63k}$ or $\frac{16}{9}$.

2. An open pipe which has a fundamental frequency of 80 Hz is vibrating with 7 nodes between its two ends. When the note from the open pipe combines with the note from a closed pipe which is vibrating with 3 nodes between its two ends (not counting the node at one end), beats of 14 Hz are produced. Give the possible values of the fundamental frequency of the closed pipe. When the closed pipe is slightly shortened, the beat frequency increases. Explain how the fundamental frequency of the closed pipe can be determined by the change in the beat frequency. Calculate the length of the open pipe if the length of the closed pipe before it was shortened was d cm.

   Answer: The open pipe has 7 nodes so it is at its 7th harmonic and its frequency is given by 80 Hz times 7 i.e. 560 Hz. Since the beat frequency is 14 Hz, the frequency of the closed pipe is either 560 Hz minus 14 Hz i.e. 546 Hz, or 560 Hz plus 14 Hz i.e. 574 Hz. The closed pipe must be at its 7th harmonic since it has 3 nodes, and its fundamental frequency is therefore either 546 Hz divided by 7 i.e. 78 Hz, or 574 Hz divided by 7 i.e. 82 Hz. When the closed pipe is shortened, its frequency will increase, so if the beat frequency increases, the closed pipe's frequency must have moved further from that of the open pipe.

The closed pipe's frequency thus must have been initially higher than that of the open pipe, i.e. it was equal to 574 Hz, and its fundamental frequency is therefore 82 Hz. An open pipe of the same length would have a fundamental frequency double this i.e. 164 Hz, and hence the length of the open pipe is given by d cm times $\frac{164}{80}$ i.e. $\frac{41d}{20}$ cm.

3. According to one theory of consonance or dissonance, the degree of consonance between any two notes depends on the number of harmonics of one note which coincide with the harmonics of the other note. Use this theory of consonance and dissonance to compare the consonance of a 60 Hz note with a second note which is higher by each of the following intervals. (You need only consider the first 18 harmonics of the 60 Hz note for the comparison.)

   (a) A Just third.

   (b) A Just fourth.

   (c) A Just fifth.

   (d) A Just seventh.

   Answer: The first 18 harmonics of the 60 Hz note are: 60 Hz, 120 Hz, 180 Hz, 240 Hz, 300 Hz, 360 Hz, 420 Hz, 480 Hz, 540 Hz, 600 Hz, 660 Hz, 720 Hz, 780 Hz, 840 Hz, 900 Hz, 960 Hz, 1,020 Hz and 1,080 Hz. All the harmonics of the higher note for each of the above intervals are listed below, with the harmonics which will coincide with a harmonic of the 60 Hz note highlighted in **bold**:

(a) A Just third above 60 Hz equals 60 Hz times $\frac{5}{4}$ i.e. 75 Hz. Harmonics: 75 Hz, 150 Hz, 225 Hz Hz, **300 Hz** Hz,375 Hz, 450 Hz, 525 Hz, **600 Hz**, 675 Hz, 750 Hz, 825 Hz, **900 Hz**, 975 Hz and 1050 Hz.

(b) A Just fourth above 60 Hz equals 60 Hz times $\frac{4}{3}$ i.e. 80 Hz. Harmonics: 80 Hz, 160 Hz, **240 Hz** Hz, 320 Hz, 400 Hz, **480 Hz** Hz, 560 Hz, 640 Hz, **720 Hz** Hz, 800 Hz, 880 Hz, **960 Hz** Hz and 1,040 Hz.

(c) A Just fifth above 60 Hz equals 60 Hz times $\frac{3}{2}$ i.e. 90 Hz. Harmonics: 90 Hz, **180 Hz** Hz, 270 Hz, **360 Hz** Hz, 450 Hz, **540 Hz** Hz, 630 Hz, **720 Hz**, 810 Hz, **900 Hz** Hz, 990 Hz and **1,080 Hz** Hz.

(d) A Just seventh above 60 Hz equals 60 Hz times $\frac{15}{8}$ i.e. 112.5 Hz. Harmonics: 112.5 Hz, 225 Hz, 337.5 Hz, 450 Hz, 562.5 Hz, 675 Hz, 787.5 Hz, **900 Hz** and 1,012.5 Hz.

The most consonant is the Just fifth with six harmonics coinciding and the next most consonant is the Just fourth with four harmonics coinciding. The next most consonant is the Just third with three harmonics coinciding and the least consonant is the Just seventh with only one harmonic coinciding.

4. A Cristofori piano has an action for each of its keys having three levers which cause the corresponding hammer to move upwards to strike the corresponding string when the key is struck downwards. The

upwards velocity of the hammer is equal to the movement of the downwards key multiplied by the first, second and third levers by factors of 1, 1.6 and 4.5 times respectively. Calculate the upwards velocity of the hammer when the downwards velocity of the corresponding key is 2.5 cm per second. The third lever of the action is then replaced with a new lever which has a multiplication factor different from 4.5 times. After the new lever has been installed, a downwards velocity of the key of 2.4 cm per second is required to give the same upwards velocity of the hammer as before. What is the multiplication factor of the new third lever?

Answer: The three levers when acting together will have a combined multiplication factor equal to 1 times 1.6 times 4.5 i.e. 7.2. Therefore the upwards velocity of the hammer is equal to 2.5 cm per second times 7.2 i.e. 18.0 cm per second. A downwards key velocity of 2.4 cm per second is required for the same upwards velocity of the hammer after the new third lever has been installed. Hence the new combined multiplication factor for the three levers acting together is equal to 18.0 cm per second divided by 2.4 cm per second i.e. 7.5. We can thus deduce that the multiplication factor of the new third lever is equal to 7.5 divided by 1.6, giving a multiplication factor of 4.6875 times for the new third lever.

5. On a particular grand piano, the soft (left) pedal, the sostenuto (middle) pedal and the sustain (right) pedal function normally. A pianist then performs on

the piano, and each of the following four different situations regarding the pedals may occur. In situation 1, the notes D2 and A2 are depressed, then the sostenuto pedal is depressed, and then the D2 and A2 keys are released while the sostenuto pedal remains depressed. Will the notes D2 and A2 be sustained? In situation 2, the sostenuto pedal is depressed, then the notes D2 and A2 are depressed, and then these keys are released. If she keeps pressing the sostenuto pedal will the notes D2 and A2 be sustained? In situation 3, the the keys D2 and A2 are depressed, then the sostenuto pedal is depressed and then the keys D2 and A2 are released, and then the sostenuto pedal is released. Will the D2 and A2 notes be sustained in this case? In situation 4, the sustain pedal is depressed, then the keys D2 and A2 are depressed and then released, keeping the sustain pedal depressed. Will the D2 and A2 notes be kept sustained?

Answer: If the sostenuto pedal is depressed after the keys have been depressed, after the keys are released, the notes D2 and A2 will be sustained as long as the sostenuto pedal is kept depressed. If the sostenuto pedal is depressed before the keys have been depressed, this will not happen. If the sostenuto pedal is released, the notes will not be sustained. Therefore the notes will be sustained only in situation 1 but not in situations 2 and 3. If the sustain pedal is depressed before or while the keys D2 and A2 are depressed, the notes D2 and A2 will be sustained as long as the sustain pedal is depressed. Therefore the notes D2 and A2 will be sustained in situation 4.

Scientific Inquiry discussion points

(a) It has now been ascertained, by making scientific observations and from the technical knowledge of how the piano action works, that a pianist playing a grand piano has only one possible effect on the sound produced when he or she strikes a piano key. All the pianist can do is to impart a certain downwards velocity to a piano key, which the mechanical leverage of the piano action converts to a faster upwards velocity of the corresponding hammer to strike the corresponding string. Hence the pianist can only affect the loudness of the sound produced. However, many pianists use their arms, hands and fingers in ways which they believe can also affect other aspects of the sound produced, such as tone quality, even though this is not the case. This is thus an example of public understanding and perception which does not correspond to the actual scientific facts. Can you think of other similar examples in everyday life?

One such example is an early belief that we live on a flat earth, as our visual perception of the surface of the earth which we can see appears to be that of a flat surface. While the earth is of course round, the great size of the earth in relation to our physical view of it seems to suggest a flat earth. Of course more careful observations of the horizon as well as photographs of the earth from space give incontro-

vertible proof of the earth's roundness. A more recent and current example is climate change. While the scientific evidence for the increasing amount of carbon dioxide in the earth's atmosphere being due to man-made causes has become strong enough to be accepted by the great majority of the scientific community and most of society, a small but significant proportion of the public and policy makers still deny that the causes of climate change are due to humans. In an issue like climate change, the understanding and perception of the public may be crucial to society's ability to deal with the undesirable effects of this phenomena in a timely and adequate fashion.