PC5215, Numerical Recipes with Applications

Lab 1, due Monday, 9 September 2024

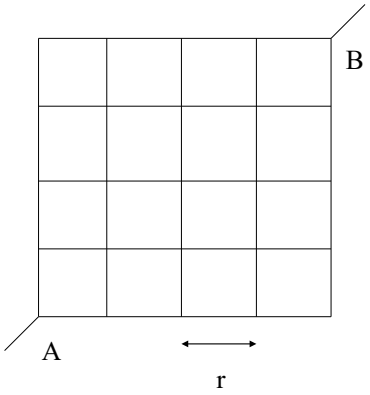
Submit your codes in PDF format at Canvas with a report of the steps, analysis, and *answers. It is not acceptable to submit just the codes without running data or missing English descriptions.*

1. Use the Python version of the "Numerical Recipes" LU decomposition functions [ludcmp(), lubksb()] to solve the following 4×4 linear equation, and verify, by hand or using other software, such as Matlab, or a consistent check, that the answers are correct. Thus, the code is checked bug-free.

$$\begin{bmatrix} 1 & 3 & 3 & -5 \\ 2 & -4 & 7 & -1 \\ 7 & 1/2 & 3 & -6 \\ 9 & -2 & 3 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 3 \\ -10 \end{bmatrix}$$

2. Consider square grids of $L \times L$ resistor network (shown below on the next page for the case of L=4). Given that point A has a voltage of 0 volt and B a voltage of 1 volt, compute the total current flowing from B to A, in units of ampere. We assume that each link has a resistance r = 1 Ω. Use the same linear solver as in problem 1 above. Report the results on L = 1, 2, 4, 8, 16, 32, …, in the power of two (as large as you can get), and the times needed (using Python time()). Then, compare the currents and CPU times with scipy linalg.solve() results as well. Also, check your computer answer against hand calculation for the case of L= 2. It would be helpful if you could tablet the results as a table.

To set up the linear equations, you need (1) Ohm's law $I_{ij} = (V_j - V_i)/r$ for each link, (2) conservation of current $\Sigma_j I_{ij} = 0$ at each node. And form linear equations for the voltage at each node. You need a good naming convention for the sites in order to construct matrix A, and figure out what is b, such that $Ax=b$, where x_i is the voltage of each node. Pay attention to the difference between the internal node, which has four neighbors, and those at the edge, which has fewer neighbors. Be careful about the sign of the current. The program should work for general L as an input parameter to a function defined via def.


B
A
r