GEK1536, Computation and Machine, Tutorial 5
(For week 8 starting 27 Feb 06)

1. Write the number 'two thousand and six' in Egyptian, Babylonian, Maya, Attic, Greek letter, Roman, Chinese rod, modern Hindu-Arabic, and binary forms. Can you also try Hebrew?

2. (a) Show that if a number written in decimal fraction is periodic, e.g., 3.123123123123123123…, 4.222222222222, or 8.743431300000000000, then it is a rational number, i.e., the number can be rewritten as ratio of two integers, p/q. (b) Conversely, if it is a rational number, then in decimal expansion, it will eventually become periodic. Show this by examples, and also by general argument. Thus irrational numbers are non-periodic in the decimal expansions.

3. Following a similar idea that proved that $\sqrt{2}$ is an irrational number, prove that $\sqrt{3}$ is also an irrational number.

4. Compute approximations of π using inscribed n-sided polygons (that is, a regular polygon of equal sides inside a circle, with vertices just touch the circle) – the Archimedes way. What is your estimate if it is a square (n=4), hexagon (n=6) and 12-sided polygon (n=12)?

Home Work (*hand in the following week tutorial*)

5. (**Homework**) Complex numbers are useful in many calculations. Euler showed an interesting relationship between complex exponential and sine and cosine functions – the Euler formula:

$$e^{i\alpha} = \cos(\alpha) + i\sin(\alpha), \quad i = \sqrt{-1},$$

where α is a real number, $\cos(\alpha)$ is the exponential's real part, and $\sin(\alpha)$ is its imaginary part. (a) Show a graph of the sine and cosine functions for the angle from 0 to 2π (That is the function value on the vertical axis and α on the horizontal axis). (b) Draw the curve traced out by the complex number $e^{i\alpha}$ on the complex plane as α varies from 0 to 2π (imaginary part on the vertical axis and real part on horizontal axis). (c) Using the Euler formula, show that

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta).$$

You need to use the fact about exponents, $a^x a^y = a^{x+y}$.

6. (**Homework**) There are many interesting formulas associated with π, such as

(a) $\pi = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \cdots,$

(b) $\pi = \frac{16}{5} - \frac{4}{239} - \frac{16}{3\cdot 5^3} + \frac{16}{5\cdot 5^5} - \frac{16}{7\cdot 5^7} + \cdots,$

(c) $\frac{4}{\pi} = 1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{2 + \cdots}}}}$

(d) $\frac{1}{\pi} = \frac{\sqrt{8}\cdot 1103}{9801} + \frac{\sqrt{8}\cdot 4!(1103 + 26390)}{9801\cdot 396^4} + \frac{\sqrt{8}\cdot 8!(1103 + 26390\cdot 2)}{9801\cdot 2^4\cdot 396^8} + \cdots$

(e) Let $y_0 = \sqrt{2} - 1, \quad a_0 = 6 - 4\sqrt{2}, \quad \pi_1 = 1/a_0;$

$$y_1 = \frac{1-(1-y_0{}^4)^{1/4}}{1+(1-y_0{}^4)^{1/4}},\ a_1 = a_0(1+y_1)^4 - 2^3 y_1(1+y_1+y_1{}^2), \quad \pi_2 = 1/a_1;$$

$$y_2 = \frac{1-(1-y_1{}^4)^{1/4}}{1+(1-y_1{}^4)^{1/4}},\ a_2 = a_1(1+y_2)^4 - 2^5 y_2(1+y_2+y_2{}^2) \quad \pi_3 = 1/a_2;$$

Use a calculator to compute π according to (a) to (e); make a table of results tabulating for each formula, and for each extra term used, like this:

Formula	First term only	First + second	First + second + 3rd	1^{st} + 2^{nd} + 3^{rd} + 4th	1^{st} + 2^{nd} + 3^{rd} + 4^{th} + 5th
(a)					
(b)					
(c)					
(d)					
(e)	π_1	π_2	π_3		

For (d) and (e), you need only go up to the first three terms. For the rest, you can go up to the fifth term. Conclude, from the above calculations, which method(s) are best for accuracy. Why the answers for (a) and (c) are the same?