# GEK1536, Computation and Machine, Tutorial 8

(For week 11 starting 20 Mar 06)

1. To subtract with Pascal's machine, one need to add its 10's complement. Find the 10's complement of the following 6-digit numbers:
(a) 000001, (b) 023957, (c) 231004, (d) 521227, (e) 871226.
Which of the above numbers should be interpreted as representing negative values?

2. In logical circuit design, it is sufficient if we know how to build (1) logical AND, (2) logical OR, and (3) Negation circuits (they are called logical gates). See the figure below. Use only these three types of devices to build a one-bit adder. Note that a one-bit adder has two inputs (a, and b) and two outputs, a result bit d and a carry bit c, with the following effect:

a	b	c	d
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0
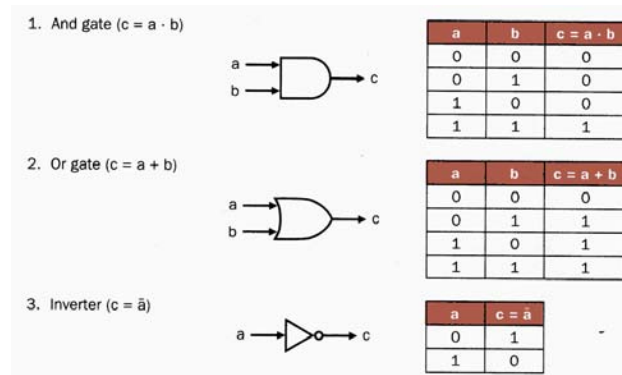
3. (a) Draw a diagram (similar to that on lecture 8, slide 30) to show the rules for a Turing machine for detecting palindrome. This Turing machine is
   states [1=START,2,3,4,5,6,7,8=HALT]
   characters [a,b,Δ]
   program:
   (1,a,2,Δ,R), (1,b,5,Δ,R), (1,Δ,8,Δ,R), (2,a,2,a,R), (2,b,2,b,R), (2,Δ,3,Δ,L), (3,a,4,Δ,L), (3,Δ,8,Δ,R), (4,a,4,a,L), (4,b,4,b,L), (4,Δ,1,Δ,R), (5,a,5,a,R), (5,b,5,b,R), (5,Δ,6,Δ,L), (6,Δ,8,Δ,R), (6,b,7,Δ,L), (7,a,7,a,L), (7,b,7,b,L), (7,Δ,1,Δ,R).
   (b) Input the following strings to see the execution of the machine: (1) "ab Δ Δ Δ…" (1) "aabaa Δ Δ Δ…". Convince yourself that the machine indeed crashes unless the input is a palindrome.

4. (**Homework**) Design a Turing machine that does binary subtraction by 1. The characters should be only "0", "1" and space "Δ".   The machine can not handle negative numbers. If the result turns out negative, the machine crash.   The Turing machine can be specified either as a list of 5 items, or as a diagram of the states. The effect of the machine on tape is like this: e.g., if the input string is
"0 0 1 Δ Δ Δ …"
The result is
"1 1 Δ Δ Δ Δ …".
The binary number is read backwards.

5. (**Homework**) Design a Turing machine to do binary addition of two arbitrary binary numbers (all zero or positive). After the computation, the input is erased and the tape is replaced by the answer. For example, if the input string is
"0 0 1 1 0 1 Δ 1 1 0 1 Δ Δ Δ Δ…"
the contents on the tape are
"ΔΔΔΔ ΔΔ Δ Δ ΔΔΔ Δ 1 1 1 0 1 1 ΔΔΔ…"
after the machine halts. Notice that we read the binary numbers backward (least significant bit first).   One extra symbol, say, '●', may be necessary for accomplishing the operation.

This is a difficult problem. Extra credit will be given.