

PC2135 Thermodynamics and Statistical Mechanics, homework 1

Due 25 Jan Wednesday 11:59PM.

Instructions for answering questions. Write down steps and thinking processes, which are as important as the final answers. The question numbers refer to Daniel V. Schroeder, “an introduction to thermal physics”, Oxford (2021). Upload a single PDF file at Canvas at Assignments -> homework 1 using your student ID as file name, e.g., A0212330X-homework-1.pdf. If it is handwritten, preferably use a digital pen, otherwise use scan software instead of taking jpeg pictures.

Problems 1.2 (10 marks), 1.16 (a), (b), (c) (30 marks), 1.24 (10 marks), 1.34 (a) (b) (c) (30 marks)

Problem 1.2. The Rankine temperature scale (abbreviated $^{\circ}\text{R}$) uses the same size degrees as Fahrenheit, but measured up from absolute zero like kelvin (so Rankine is to Fahrenheit as kelvin is to Celsius). Find the conversion formula between Rankine and Fahrenheit, and also between Rankine and kelvin. What is room temperature on the Rankine scale?

Problem 1.16. The exponential atmosphere.

- (a) Consider a horizontal slab of air whose thickness (height) is dz . If this slab is at rest, the pressure holding it up from below must balance both the pressure from above and the weight of the slab. Use this fact to find an expression for dP/dz , the variation of pressure with altitude, in terms of the density of air.
- (b) Use the ideal gas law to write the density of air in terms of pressure, temperature, and the average mass m of the air molecules. (The information needed to calculate m is given in Problem 1.14.) Show, then, that the pressure obeys the differential equation

$$\frac{dP}{dz} = -\frac{mg}{kT}P,$$

called the **barometric equation**.

- (c) Assuming that the temperature of the atmosphere is independent of height (not a great assumption but not terrible either), solve the barometric equation to obtain the pressure as a function of height: $P(z) = P(0)e^{-mgz/kT}$. Show also that the density obeys a similar equation.

Problem 1.24. Calculate the total thermal energy in a gram of lead at room temperature, assuming that none of the degrees of freedom are “frozen out” (this happens to be a good assumption in this case).

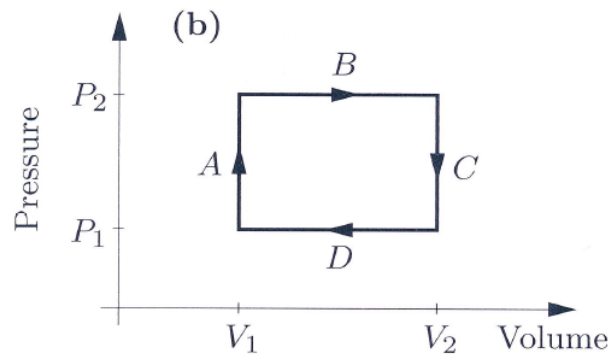


Figure 1.10. PV diagrams for Problems 1.33 and 1.34.

Problem 1.34. An ideal diatomic gas, in a cylinder with a movable piston, undergoes the rectangular cyclic process shown in Figure 1.10(b). Assume that the temperature is always such that rotational degrees of freedom are active, but vibrational modes are “frozen out.” Also assume that the only type of work done on the gas is quasistatic compression-expansion work.

- (a) For each of the four steps A through D , compute the work done on the gas, the heat added to the gas, and the change in the energy content of the gas. Express all answers in terms of P_1 , P_2 , V_1 , and V_2 . (Hint: Compute ΔU before Q , using the ideal gas law and the equipartition theorem.)
- (b) Describe in words what is physically being done during each of the four steps; for example, during step A , heat is added to the gas (from an external flame or something) while the piston is held fixed.
- (c) Compute the net work done on the gas, the net heat added to the gas, and the net change in the energy of the gas during the entire cycle. Are the results as you expected? Explain briefly.