PC2135 Thermodynamics and Statical Mechanics, homework 6 (last set)

Due Tuesday 18 Apr 11:59PM

Problems 7.8 (*28 marks (a) to (d), 7 marks each*). Problem 7.26 (*24 marks, each subquestion is 8 marks for the rest*). Problem 7.43 (*24 marks). 7.44* (*24 marks*). Total 100 marks.

Problem 7.8. Suppose you have a "box" in which each particle may occupy any of 10 single-particle states. For simplicity, assume that each of these states has energy zero.

(a) What is the partition function of this system if the box contains only one particle?

(b) What is the partition function of this system if the box contains two distinguishable particles?

(c) What is the partition function if the box contains two identical bosons?

(d) What is the partition function if the box contains two identical fermions?

Problem 7.26. In this problem you will model helium-3 as a noninteracting Fermi gas. Although ^{3}He liquefies at low temperatures, the liquid has an unusually low density and behaves in many ways like a gas because the forces between the atoms are so weak. Helium-3 atoms are spin-1/2 fermions, because of the unpaired neutron in the nucleus.

(a) Pretending that liquid ^{3}He is a noninteracting Fermi gas, calculate the Fermi energy and the Fermi temperature. The molar volume (at low pressures) is 37 cm^3.

(b) Calculate the heat capacity for $T \ll T_F$, and compare to the experimental result $C_V = (2.8\ \text{K}^{-1})NkT$ (in the low-temperature limit). (Don't expect perfect agreement.)

(c) The entropy of *solid* ^{3}He below 1 K is almost entirely due to its multiplicity of nuclear spin alignments. Sketch a graph S vs. T for liquid and solid ^{3}He at low temperature, and estimate the temperature at which the liquid and solid have the same entropy. Discuss the shape of the solid-liquid phase boundary shown in Figure 5.13.

Problem 7.43. At the surface of the sun, the temperature is approximately 5800 K.

(a) How much energy is contained in the electromagnetic radiation filling a cubic meter of space at the sun's surface?

(b) Sketch the spectrum of this radiation as a function of photon energy. Mark the region of the spectrum that corresponds to visible wavelengths, between 400 nm and 700 nm.

(c) What fraction of the energy is in the visible portion of the spectrum? (Hint: Do the integral numerically.)

Problem 7.44. Number of photons in a photon gas.

(a) Show that the number of photons in equilibrium in a box of volume V at temperature T is

$$N = 8\pi V \left(\frac{kT}{hc}\right)^3 \int_0^\infty \frac{x^2}{e^x - 1}\, dx.$$

The integral cannot be done analytically; either look it up in a table or evaluate it numerically.

(b) How does this result compare to the formula derived in the text for the entropy of a photon gas? (What is the entropy per photon, in terms of k?)

(c) Calculate the number of photons per cubic meter at the following temperatures: 300 K; 1500 K (a typical kiln); 2.73 K (the cosmic background radiation).