Singapore-MIT Alliance, CME5233 – Particle Methods and Molecular Dynamics

Tutorial 1, Monday 2:30 – 4:00, 6 Nov 2006

1. (a) Buffon needle problem. Prove the probability of the needle intersecting the equally spaced line is $P = 2L/(\pi d)$, where L is the length of the needle, and d is strip spacing, and $L < d$.
   (b) More challenging one (extra credit). If the strips are alternating a and b with $L < \min(a, b)$, what would be the answer for the intersecting probability?

2. Interesting fact. If the modulus $m = 2^e$, the low-order bits are much less random than the high-order bits in a linear congurential random number sequence. More precisely, the least significant bit is either constant or strictly alternating. The last two bits cannot have a period of more than 4; and the low-order four bits has a period of length 16 or less. To demonstrate this mathematically, let's define
   $$Y_n = X_n \bmod d,$$
   where d is a divisor of m, X_n is generated with the usual linear congruential method, $X_{n+1} = (a X_n + c) \bmod m$, show that
   $$Y_{n+1} = (aY_n + c) \bmod d.$$
   That is Y_n is also a linear congruential sequence with modulus d, multiplier a, and increment c. Explain the stated fact.

3. Sampling random variables. A random variable ξ with a uniform probability distribution in the interval [0, 1) is given. Work out a method to generate a random integer k ≥ 0 from ξ with the probability distribution
   $$P_k = (1-\alpha)\alpha^k, \quad k = 0, 1, 2, 3, \ldots$$
   where α is some constant satisfying $0 < \alpha < 1$. [Hint, use the inverse distribution function method].