Singapore-MIT Alliance, CME5233 – Particle Methods and Molecular Dynamics

Tutorial 3, Monday 2:30 – 4:00, 20 Nov 2006

1. Let the transition matrix be given as

$$W = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0.2 & 0.3 & 0 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   a. Draw a diagram with arrows and numbers to show the same information. State 1, 2, 3, and 4 correspond to the index of the matrix.
   b. Is the state 2 recurrent?
   c. Is the state 3 transient?
   d. Is the Markov chain defined by the matrix W irreducible?
   e. Is the Markov chain ergodic?

2. A transition matrix W in a Markov chain is given by

$$\begin{bmatrix} 1/2 & 0 & 1/2 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1 & 0 \end{bmatrix}.$$

   The state space is {1, 2, 3}. The matrix element W_{ij} corresponds to transition probability from state i to j, i=1,2,3, j=1,2,3. Answer the following questions:
   (a) Draw a diagram (states and transition arrows) to represent the above transition matrix.
   (b) Suppose that the system is in state 1 at the beginning (step 0), what is the probability that the system reaches state 2 in n steps for *the first time* (first passage) for n equals 1, 2, 3, and 4?
   (c) Continuing with part (b), what is the probability that the system will reach state 2, for whatever number of steps, for the first time.

3. You have three states of your mind – sleepy, excited, or bored. If you are sleepy, chances are you will be continuing sleepy with high probability, say, 0.9, and left with 0.1 probability being excited. If you are excited, you will be still excited with probability ½, and another ½ for being bored. If you are bored, with high probability of 0.8 you'll be sleepy and another 0.2 being bored.
   (a) Assuming the state of your mind is a Markov chain, write down the transition matrix W.
   (b) Assuming initially at time t=0, you are sleepy (for sure), what is the probability that you will be excited at time step t = 2? Similarly, the probability for being bored?
   (c) What is the probability distribution of states of your mind at $t \to \infty$?