CZ3272, MC & MD, Tutorial 3 (*for week 8, Wednesday 4 Oct 06*)

1. Suppose that ξ_1 and ξ_2 are two random variables uniformly distributed between 0 and 1, what is the probability distribution of the following new variables:

$$\xi_1 + \xi_2,\quad \xi_1 - \xi_2,\quad \xi_1 * \xi_2,\quad \xi_1 / \xi_2,\quad \max(\xi_1 , \xi_2),\quad \min(\xi_1 , \xi_2)?$$

(Hint: determine the cumulative distribution function $F(x)$ of the new variables, then $p(x) = \mathrm{d}F(x)/\mathrm{d}x$.)

2. Suppose we need to compute the following one-dimensional integral by simple sampling Monte Carlo:

$$\int_0^{\pi} \sin(x)\, dx$$

(a) Write down the formula for Monte Carlo estimate and formula for error estimate, assuming a uniform probability distribution for x.

(b) Use a variance reduction technique discussed in class, that is, divide the region into two parts $[0, \pi/2]$ and $[\pi/2, \pi]$, and consider triangular distribution for x instead. Show that there is indeed a variance reduction. Compute the variance in case (a) and (b), respectively. [Integrals may be computed using Matlab or Mathematica].

(c) How does one generate a triangular distribution required in (b)?

3. You have three states of your mind – sleepy, excited, or bored. If you are sleepy, chances are you will be continuing sleepy with high probability, say, 0.9, and left with 0.1 probability being excited. If you are excited, you will be still excited with probability ½, and another ½ for being bored. If you are bored, with high probability of 0.8 you'll be sleepy and another 0.2 being bored.

(a) Assuming the state of your mind is a Markov chain, write down the transition matrix W.

(b) Assuming initially at time t=0, you are sleepy (for sure), what is the probability that you will be excited at time step $t = 2$? Similarly, the probability for being bored?

(c) What is the probability distribution of states of your mind at $t \to \infty$?

(d) Describe a computer simulation of the states of mind according to the above Markov chain.

CZ3272 MC & MD, Lab 3 (*for week 6 to 8, 18 Sep to 6 Oct 2006*)

Due Friday 6 Oct 2006

1. Consider a random walk in a one-dimensional lattice so that the walker is restricted to the locations 0, 1, 2, …, j, …, $N-1$. To be definite, we take $N = 10$. The walker moves to the left or right with equal probability of 1/2. When it is at the boundary site (0 or N-1) and the walker tries to move outside the system, the move is rejected and the walker stays where it is.

(a) Write down the transition matrix W.

(b) Assuming at time $t = 0$, the walker is always at the site 0, determine the time-dependence of the probability that the walker is at site $N-1$, that is, the function $P_{N-1}(t)$, by Monte Carlo method (i.e., a direct simulation of the above Markov chain). To be accurate, you have to repeat many runs (say 10^6) and take the average. Sample t from 0 to about few hundred steps.

(c) What is the answer in the limit t goes to infinity?

(d) Can you compute $P_{N-1}(t)$ by other means without simulation? If you can, do it and compare with result in (b).