Supplementary Worksheet #1
Exchange Symmetry, Quantum Numbers, and Angular Momentum

1 Exchange Symmetry of Wavefunctions

1. What restrictions does the spin–statistics theorem impose on fermions and bosons?
2. Show that two identical fermions cannot occupy the same quantum state.
3. Consider the annihilation process
$$e^- + e^+ \to 2\gamma.$$
Why does this process challenge the usual interpretation of $|\Psi|^2$ and how is this resolved in QFT?

2 Elementary Particles

4. Complete the empty cells in the following table with the appropriate values:

Particle	s	Q	**Colour**	B	**Other labels/quantum numbers**
down (d)					
up (u)					
strange (s)					
charm (c)					
bottom (b)					
top (t)					
electron (e^-)					
electron neutrino (ν_e)					
muon (μ^-)					
muon neutrino (ν_μ)					
tau (τ^-)					
tau neutrino (ν_τ)					
photon (γ)					
vector bosons $(W^\pm, Z^0)$					
gluon (g)					

3 Quark Composition and Hadron Properties

5. Complete the table below.

Particle	Quark content	Charge	Spin	Mass	Lifetime
Proton					
π^0					
π^+					
π^-					
ρ^0					
ρ^+					
ρ^-					
Δ^+					

6. Neutral pion structure: The π^0 involve a superposition of two quark – anti-quark states. What does this mean?

7. Spin and mass: The proton and Δ^+ have the same quark content but different spins. Propose a physical reason why the higher-spin state has a larger mass.

4 Addition of Angular Momentum

8. State the theorem for the addition of two angular momenta,

$$\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2.$$

Clearly indicate the allowed values of the total angular momentum quantum number j.

9. Apply the angular momentum addition theorem to an atomic electron in a p state.
List the allowed values of j and m_j.

10. Apply the angular momentum addition theorem to the spins of two electrons.

a) List the allowed values of j and m_j.

b) Write down the coupled states using $|j, m_j\rangle$ notation.