Supplementary Worksheet #2
A bit of Relativity and A lot Angular Momentum

1 Relativity and Kinematics

1. When must special relativity be incorporated into a physical analysis? State a clear rule of thumb.

2. For each of the following processes, determine whether it can occur.
If it can, state the necessary **kinematic** condition and explain your reasoning.

a) $A \to B$ with $m_A > m_B$

b) $A \to B$ with $m_A < m_B$

c) $A \to B + C$ with $E_A > m_B + m_C$

3. Single-particle processes: Can the following occur in free space?
In each case, justify your answer using conservation laws.

a) Photon absorption by a **free** electron: $e^- + \gamma \to e^-$

b) Pair production in **free** space: $\gamma \to e^- + e^+$

2 Addition of Angular Momentum

4. State the theorem for the addition of two angular momenta $\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2$.
Clearly indicate the allowed values of the total angular momentum quantum number j.

5. Apply the angular momentum addition theorem to a system of two electrons.

a) List the allowed values of the total angular momentum quantum numbers j and m_j.

b) Write down the coupled states using $|j\, m_j\rangle$ notation.

6. Using the Clebsch–Gordan coefficients (figure 1), express the total angular momentum states in terms of uncoupled product states.

a) Using $|j\, m_j\rangle$ notation.

b) Using $\uparrow$ and $\downarrow$ notation.

7. Apply the angular momentum addition theorem to a system with $j_1 = 1$ and $j_2 = \frac{1}{2}$.

a) List the allowed values of the total angular momentum j and m_j.

b) Write down the corresponding coupled states.

8. Expand the state $|j = \frac{3}{2},\, m_j = \frac{3}{2}\rangle$ in terms of uncoupled basis states using $\uparrow$ and $\downarrow$ notation.

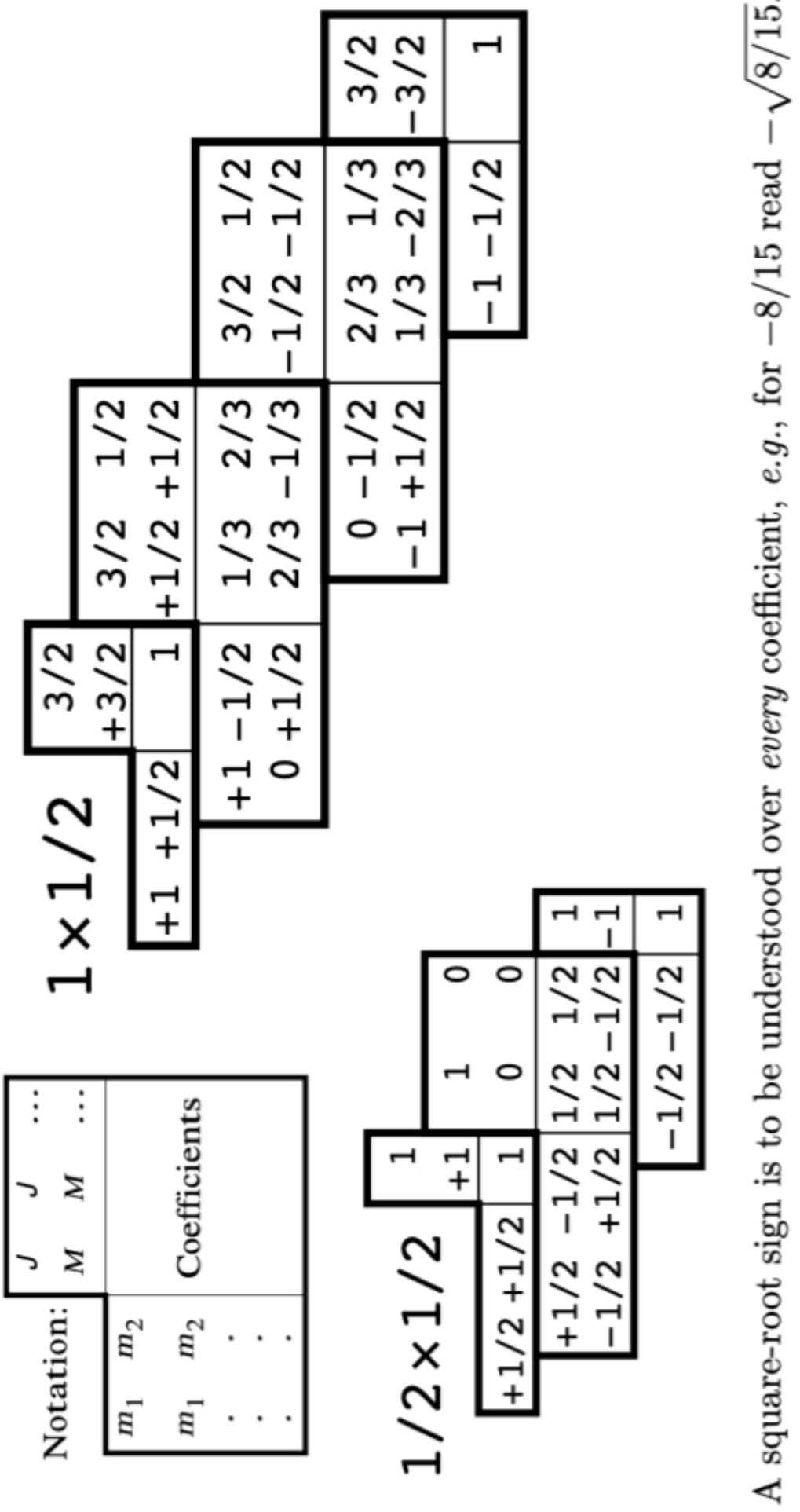


Figure 1: Clebsch Gordan Coefficients for adding angular momenta.