Supplementary Worksheet #5
Resonances, Lifetimes & Decays

1 Basics

1. Consider a two-body decay

$$A \to B + C. \tag{1}$$

Show that the quantity s_{BC},

$$s_{BC} \equiv (E_B + E_C)^2 - (\vec{p}_B + \vec{p}_C)^2 c^2, \tag{2}$$

is Lorentz invariant.

2. What is the physical significance of s_{BC} in relation to measurements of particles B and C?

3. Suggest practical ways of measuring the energies and momenta of particles B and C.

4. Explain the energy–time uncertainty relation. In your explanation, comment on how it differs conceptually from the position–momentum uncertainty relation, and clarify what is meant by Δt in this relation.

2 Lifetime and Decays

5. Suppose a particle has a constant probability $\lambda\,\mathrm{d}t$ of decaying in a short time interval $\mathrm{d}t$.

a) Show that the probability $p(t)$ that the particle survives up to time t satisfies

$$p(t) = e^{-\lambda t}.$$

b) Using $p(t)$, show that the mean (average) lifetime of the particle is

$$\langle t \rangle = \frac{1}{\lambda}.$$

c) If there are initially N_0 identical particles, show that the number remaining at time t is

$$N(t) = N_0\, e^{-\lambda t}.$$

d) The lifetime τ is often defined as the time at which the number of particles has fallen to $1/e$ of its initial value. Show that

$$\tau = \langle t \rangle = \frac{1}{\lambda}.$$

3 Unstable States and Resonances

4. Argue that the decay of an unstable particle may be described by a wave function of the form

$$\psi(\vec{r}, t) = \psi(\vec{r})\, e^{-t/2\tau}\, e^{-i\omega_0 t}. \tag{3}$$

Explain the physical meaning of each factor in this expression.

4. Show how this time dependence leads to a Breit–Wigner resonance in energy,

$$P(E)\, dE \propto \frac{dE}{(E - E_0)^2 + (\Gamma/2)^2}. \tag{4}$$

4. What is the significance of Γ?

4. How is Γ related to the energy–time uncertainty relation $\Delta E\, \Delta t \sim \hbar$?

5. What features can you highlight on figure below:

