Supplementary Worksheet #6
Parity & Angular Momentum

1 Parity

1. What is a parity transformation?
Write down how coordinates transform under a parity transformation ($\mathbf{r} \to -\mathbf{r}$), in Cartesian and spherical coordinates.

2. Under a parity transformation, how do polar vectors and axial vectors transform?
Explain why angular momentum behaves differently from position.

3. Argue that neither mechanical processes nor angular momentum can distinguish between the 'real' and 'mirror' universes.

4. Write down how the spherical harmonics ($Y_\ell^m(\theta, \phi)$) transform under a parity transformation.

5. Helicity:

a) How is helicity defined?

b) Show that helicity is a pseudoscalar.

c) Is helicity of a particle a constant?

6. State the rules governing intrinsic parity and how parities combine for composite systems.

7. Show explicitly how orbital angular momentum contributes a factor of $(-1)^\ell$ to the parity of a two-particle state.

8. Determine the intrinsic parity of the ϕ^0 given that it decays as:

$$\phi^0 \to K^+ + K^-. \tag{1}$$

State any assumptions you make about the orbital angular momentum of the final state.

9. All the reactions below correspond to a decay of the form

$$A \to B + C. \tag{2}$$

Determine which reactions are allowed and which are forbidden. If any are forbidden, indicate why.

$$0^+ \longrightarrow 0^+ + 0^- \tag{3a}$$
$$0^+ \longrightarrow 0^+ + 0^+ \tag{3b}$$
$$0^+ \longrightarrow 0^- + 0^- \tag{3c}$$
$$0^- \longrightarrow 0^+ + 0^+ \tag{3d}$$
$$0^- \longrightarrow 0^+ + 0^- \tag{3e}$$
$$0^- \longrightarrow 0^- + 0^- \tag{3f}$$

Note

- Assume that **none** of the reactions involve the weak interaction.
- I have used J^η notation where J is the total spin and η is the parity.

10. $(\tau - \theta)$ **puzzle**

a) In the 1950s, two particles τ (**not** the lepton!) and θ were found to decay via

$$\theta^+ \longrightarrow \pi^+ + \pi^- \tag{4}$$

and

$$\tau^+ \longrightarrow \pi^+ + \pi^+ + \pi^-. \tag{5}$$

Determine the intrinsic parity of the τ and θ particles.

b) It was later found that both τ and θ were the same particle, and that the above are two channels through which it can decay. What does this imply for the intrinsic parities you calculated?

11. What is the spin and parity of the photon?

2 Wave Functions and Exchange Symmetry

12. What symmetry restrictions apply to the total wave function of

a) fermions, and

b) bosons

under exchange of identical particles?

13. The total wave function of a multi-particle system may be written as a product of several components (for example, spatial, spin, and internal degrees of freedom).

For each of the following systems:

- identify the relevant components of the total wave function, and
- explain how their combined symmetry satisfies the required exchange properties.

a) Two electrons.

b) Two identical quarks.

c) A quark–antiquark pair.

d) Two neutrons.

e) A proton and a neutron.

3 Importance of Angular Momentum

14. The following reaction shows the subsequent decay into two neutrons of a "pionic deuterium" nucleus, which was formed by the absorption of a pion (π^-) by a deuteron (d):

$$\pi^- + d^+ \to n + n. \tag{6}$$

Note that:

- The spins of the pion, deuteron, and neutron are 0, 1, and $\frac{1}{2}$, respectively.
- The deuteron has even intrinsic parity.
- "Pionic deuterium" is typically formed in the s (i.e. $\ell = 0$) state.

a) Write down the possible spin states (singlet/triplet) of the combined n–n system.

b) List the possible spatial angular momentum states that can be combined with the above, while still respecting conservation of angular momentum (i.e. match total $\mathbf{J}$ of initial and final states).

c) Determine the intrinsic parity of the pion based on the above reaction, given that parity is conserved. Clearly justify each step of your reasoning.