Supplementary Worksheet #7
Cross Sections & Radiation–Matter Interactions

1 Total Cross Section

1. Give an experimental definition of the cross section σ. What does a larger cross section imply?

2. A beam containing N_0 neutrons enters a uniform material. The material contains n scattering centres per unit volume, and each scattering centre has a total cross section σ.

Assuming that the scattering centres act independently and that n and σ are constant, find the number of neutrons $N(x)$ that travel a distance x without interacting.

2 Solid Angle

3. Define the solid angle $d\Omega$ subtended by a small area dA located a distance R from a point. What is the maximum possible value of Ω?

4. A circular ring is a distance R from a point source and lies between the polar angles θ and $\theta + d\theta$.

a) Show that the solid angle subtended by the ring is

$$d\Omega = 2\pi \sin\theta \, d\theta.$$

b) Hence, verify that the maximum solid angle is

$$\Omega_{\max} = 4\pi.$$

3 Partial & Differential Cross Sections

5. Explain the difference between a **partial** cross section and a **differential** cross section.

6. Consider the Rutherford differential cross section:

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 e^2}{16\pi\epsilon_0 E}\right)^2 \frac{1}{\sin^4(\theta/2)}. \tag{1}$$

a) What is the physical meaning of $d\sigma/d\Omega$? What does the Rutherford formula tell us, and how can it be used experimentally?

b) Why is Rutherford backscattering particularly sensitive to elements with large atomic number Z?

c) Why does the differential cross section increase strongly at small scattering angles?

4 Radiation–Matter Interactions

7. Construct a tree diagram that classifies radiation–matter interactions.

5 Charged Particles and the Bragg Peak

8. What is **stopping power**? What does it tell us physically?

9. What is the utility of defining the **mass** stopping power?

10. On what does stopping power depend? Identify the most significant parameters that affect it.

$$\frac{dE}{dx} = \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{4\pi z^2 N_A Z\rho}{m_e c^2 \beta^2 A}\left[\ln\left(\frac{2m_e c^2\beta^2}{I}\right) - \ln\left(1-\beta^2\right) - \beta^2\right]$$

11. Sketch the general shape of the energy loss profile for a charged particle in matter. Indicate the region where the Bethe model is valid.

12. What is the Bragg peak? What is its physical significance?

13. Why is bremsstrahlung negligible for heavy charged particles but important for electrons?

14. When does radiative energy loss become more significant than collisional energy loss for electrons?

6 Photon Interaction Mechanisms

15. Compare the various interaction mechanisms available to a photon in matter.

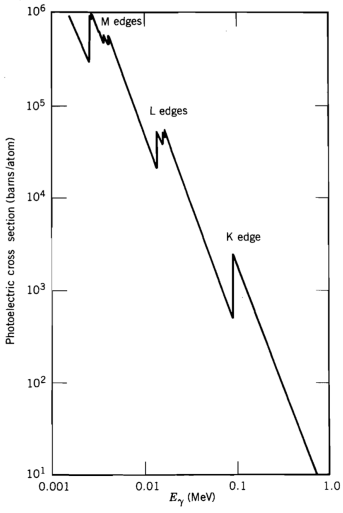
16. Explain the occurrence of the K, L, and M absorption edges.

17. Why must pair production:

a) have a threshold energy of at least $2m_e c^2$?

b) not occur in free space?

18. Why are **mass** attenuation coefficients often more useful than **linear** attenuation coefficients?



7 Rutherford Backscattering Spectrometry (RBS)

Rutherford Backscattering Spectrometry (RBS) is a materials analysis technique that uses energetic ions (typically α particles) and measures their backscattered energies to determine composition and depth structure. RBS was even used, for example, in the Surveyor VI lunar mission.

19. The RBS spectrum below was obtained using $2.2\,\text{MeV}$ α particles incident on a thin film (C, O, Fe, Mo, Au) deposited on a silicon substrate.

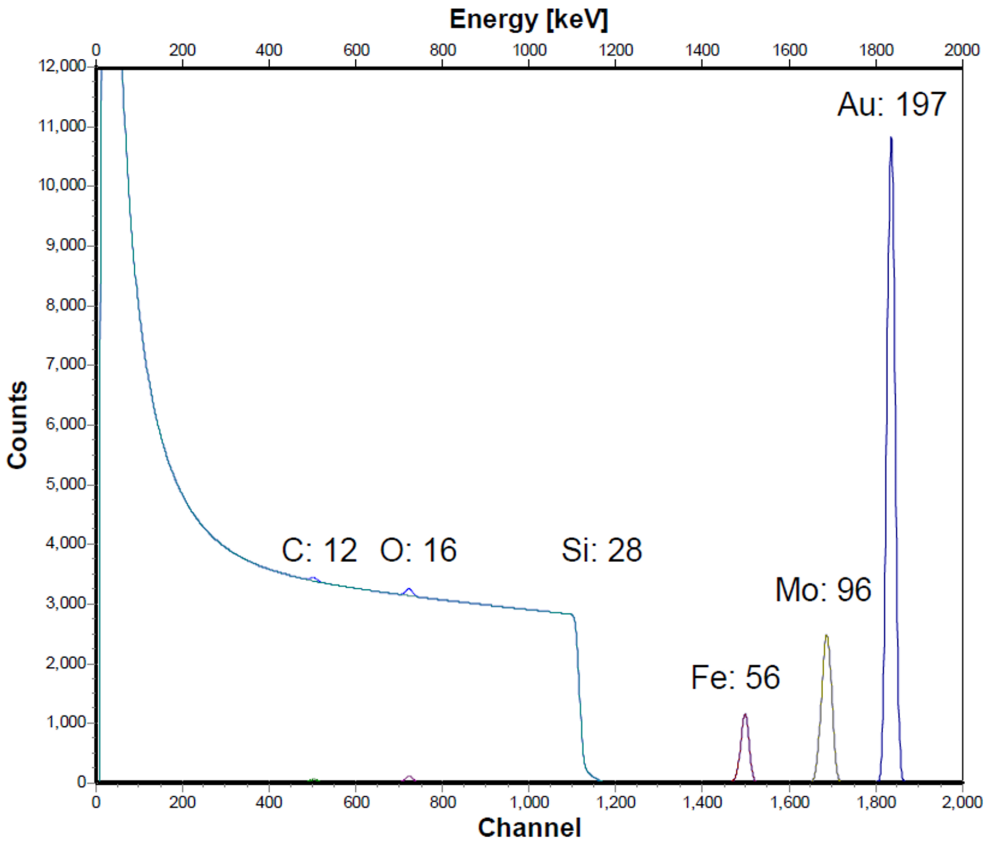
Using your understanding of two-body elastic scattering kinematics, differential cross sections, and energy loss in matter, explain the following features:

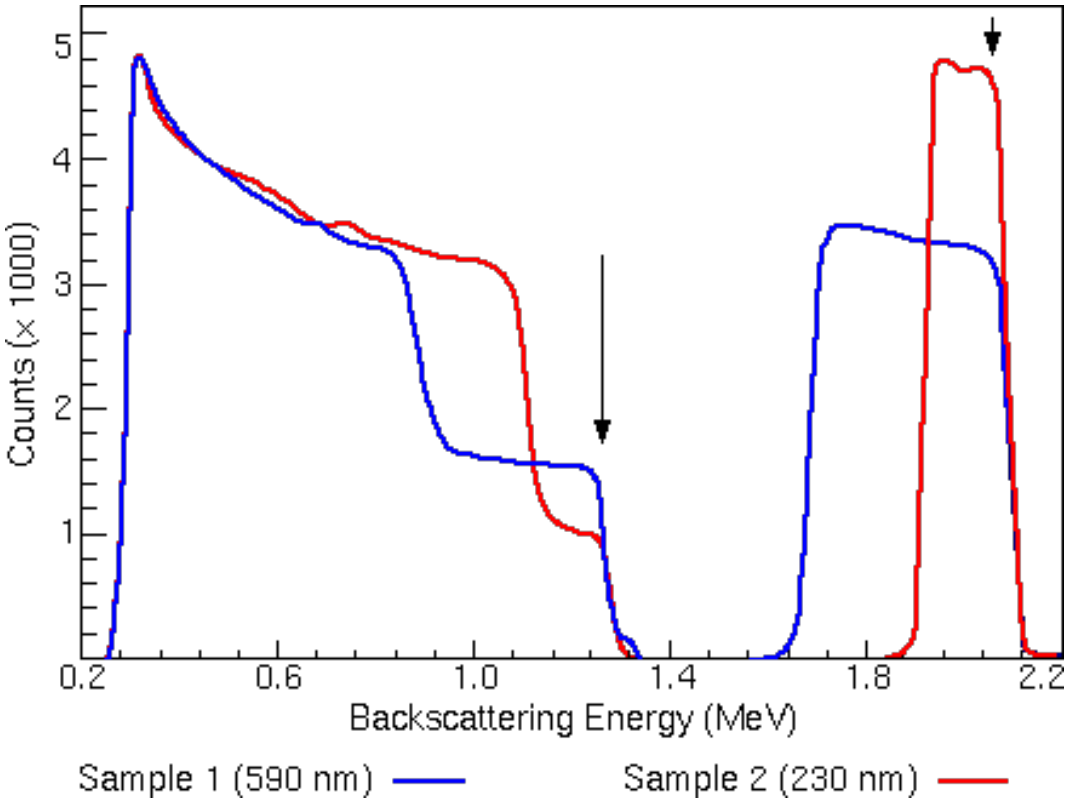
a) Why heavier elements appear at higher detected energies.

b) Why each element produces a “box-like” structure rather than a sharp line.

c) Why the width of each feature contains depth information.

d) Why the silicon substrate appears as a broad underlying structure.





20. Shown above is another set of RBS spectra for two thin films of tantalum silicide (TaSi) on a silicon substrate. Explain the shape of the spectrum.

8 Compton Scattering

21. The kinetic energy transferred to the electron in Compton scattering can be shown to be:

$$T_e = \frac{E_\gamma^2(1-\cos\theta)}{m_e c^2 + E_\gamma(1-\cos\theta)}$$

a) For what value of θ is T_e maximised?

b) Calculate the maximum energy that a $4.44\,\text{MeV}$ photon can transfer to an electron.

9 Gamma Spectroscopy: Am–Be Source

22. What are the possible interactions that a gamma photon can have in a material? How does it deposit energy?

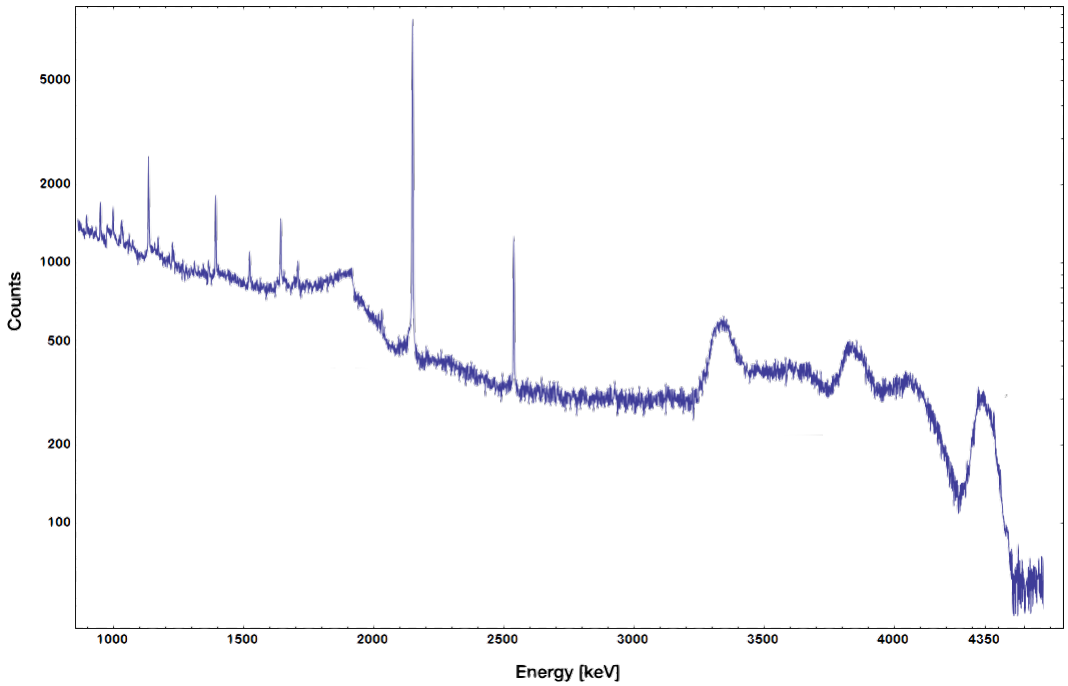
23. If a $4.44\,\text{MeV}$ photon produces an electron–positron pair and:

- one annihilation photon escapes, what energy is deposited?
- both annihilation photons escape, what energy is deposited?

24. An Am–Be source is a typical neutron source that works via:

$$^{241}\text{Am} \to \alpha + {}^{237}\text{Np} + \gamma\,(55.5\,\text{keV})$$
$${}^{9}\text{Be} + \alpha \to {}^{12}\text{C}^* + n$$
$${}^{12}\text{C}^* \to {}^{12}\text{C} + \gamma\,(4.44\,\text{MeV})$$

Shown below, is the gamma spectrum from an Am–Be source.



Using your understanding of how photons interact with matter, explain the features of the seen in the spectrum. In particular, highlight the following:

a) Full (photo-)energy peak

b) Compton continuum

c) Compton edge

d) Single escape peak

e) Double escape peak