Supplementary Worksheet #8
Size & Shape of the Nucleus

1 General Properties of the Nucleus

1. What is binding energy? What is a representative value to remember?
2. How is binding energy related to nuclear stability.
3. What are magic numbers, and why are they important?
4. When discussing the extent of the nucleus:
   a) What is a good measure/indicator of the size of a nucleus?
   b) Does this quantity describe the distribution of **charge** or **nuclear matter**?
   c) What is a more realistic distribution of charge and matter within a nucleus?

2 Probing Nuclear Size

5. What are the differences between using:
   a) α particles,
   b) electrons, and
   c) neutrons as scattering probes?

   What information can each provide?
6. Why do diffraction effects appear in scattering experiments used to probe nuclei?

3 Revisiting Cross Sections

7. Show that the change in momentum of an incoming particle that is elastically scattered is

$$\Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i,$$

   and that its magnitude is

$$|\Delta\mathbf{p}| = 2p \sin\left(\frac{\theta}{2}\right).$$

8. In the non-relativistic approximation, use $E_{\text{kin}} = \dfrac{p^2}{2m}$, express the Rutherford cross section in terms of the momentum transfer $|\Delta\mathbf{p}|$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{Zze^2}{4\pi\epsilon_0}\right)^2 \left(\frac{1}{4E}\right)^2 \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}.$$

9. How can this scattering process be described quantum mechanically using plane waves?
10. Rewrite the Rutherford cross section in terms of the wavevector transfer $\mathbf{q}$, defined by

$$\mathbf{q} = \mathbf{k}_f - \mathbf{k}_i, \quad \text{with} \quad \hbar\mathbf{q} = \Delta\mathbf{p}.$$

    What does expressing the cross section in terms of $\mathbf{q}$ tell us physically?

4 Nuclear Form Factor

11. What is the significance of using **elastically scattered electrons** to probe nuclei?

12. What is the First Born approximation?

13. Show that the scattering amplitude can be written as

$$f(\mathbf{q}) \propto \langle \psi_f | V | \psi_i \rangle = \int V(\mathbf{r})\, e^{-i\mathbf{q}\cdot\mathbf{r}}\, \mathrm{d}^3\mathbf{r}.$$

14. What is the significance of the $f(\mathbf{q})$?

15. What would be a typical procedure to extract information about the charge density ρ_{charge} of a nucleus?

5 Multipole Expansions

16. What are multipole expansions in electrostatics? What physical insight do they provide?

17. Write down the expressions for the following electric momenets and explain what information they convey:

a) the monopole ($E0$),
b) dipole ($E1$), and
c) quadrupole ($E2$) terms.

18. A stationary nuclear state has definite parity.

a) Explain what this means?
b) Show that its charge density satisfies

$$\rho(-\mathbf{r}) = \rho(\mathbf{r}).$$

c) Explain why the electric dipole operator changes sign under $\mathbf{r} \to -\mathbf{r}$.
d) Hence, show that the permanent electric dipole moment vanishes.

19. Consider an axially symmetric **intrinsic** nuclear charge distribution. Explain why its intrinsic quadrupole moment is:

a) positive for prolate nuclei,
b) zero for spherical nuclei,
c) negative for oblate nuclei.