Supplementary Worksheet #10
Spin and Parity of the Nucleus: The Shell Model

1 Parity of the Nucleus

1. What are the intrinsic parities of a proton and a neutron?

2. Consider a nuclear system with a proton in an orbital angular momentum state ℓ_p and a neutron in a state ℓ_n.

a) How can the proton and neutron have different orbital angular momentum quantum numbers?

b) Write down the total parity of the system.

3. Show that, for a system of nucleons:

a) states with an even number of particles in a given orbital ℓ_i contribute **positive parity**;

b) states with an odd number of particles contribute a factor $(-1)^{\ell_i}$.

2 Angular Momentum

4. Consider a single nucleon with orbital angular momentum ℓ and intrinsic spin $s = \frac{1}{2}$. Show that the total angular momentum of this nucleon is always a half-integer value j.

5. Now consider a system of multiple particles, each with orbital angular momentum ℓ_i and spin $s_i = \frac{1}{2}$.

a) For $N = 2$ particles, what are the possible total angular momentum values?

b) For $N = 3$ particles, what are the possible total angular momentum values?

6. What is the difference between LS **coupling** and jj **coupling**? When is each scheme more appropriate in atomic and nuclear systems?

7. What angular momentum states are obtained when an orbital angular momentum ℓ is combined with a spin $s = \frac{1}{2}$? What are their degeneracies? Complete the table below.

Orbital angular momentum state	Possible total (j) values	m_j	Degeneracy
s ($\ell = 0$)			
p ($\ell = 1$)			
d ($\ell = 2$)			
f ($\ell = 3$)			

3 Trends in Nuclear Spin

8. Show that the spin of a nucleus (I) must be:

- an integer for nuclei with an **even** number of nucleons;
- a half-integer for nuclei with an **odd** number of nucleons.

9. Find examples of nuclear spin values for the following cases:

Criterion	Example nuclide	Z, N	I
Odd Z, odd N			
Even Z, odd N			
Odd Z, even N			
Even Z, even N			

10. Based on the table above:

a) argue that there is **pairing** between nucleons;

b) argue that protons and neutrons pair separately.

4 Spin–Orbit Coupling

11. A nucleon has orbital angular momentum $\mathbf{L}$ and intrinsic spin $\mathbf{S}$, with $s = \frac{1}{2}$.

Suppose the Hamiltonian contains a spin–orbit term of the form

$$V_{so} = C\,\mathbf{L}\cdot\mathbf{S}.$$

a) Explain the physical meaning of this interaction.

b) Show that the two states $j = \ell + \frac{1}{2}$ and $j = \ell - \frac{1}{2}$ are split in energy, and that the splitting is proportional to $(2\ell+1)\hbar^2$.

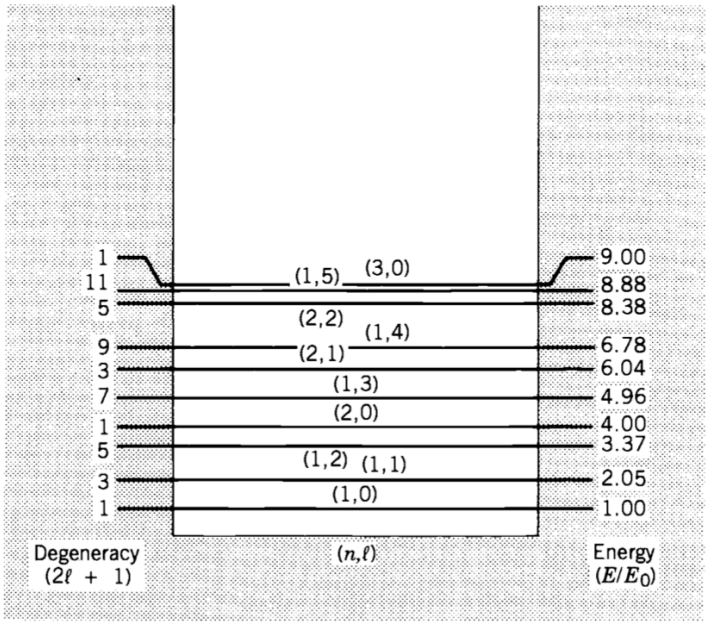
5 3D Spherical Potential

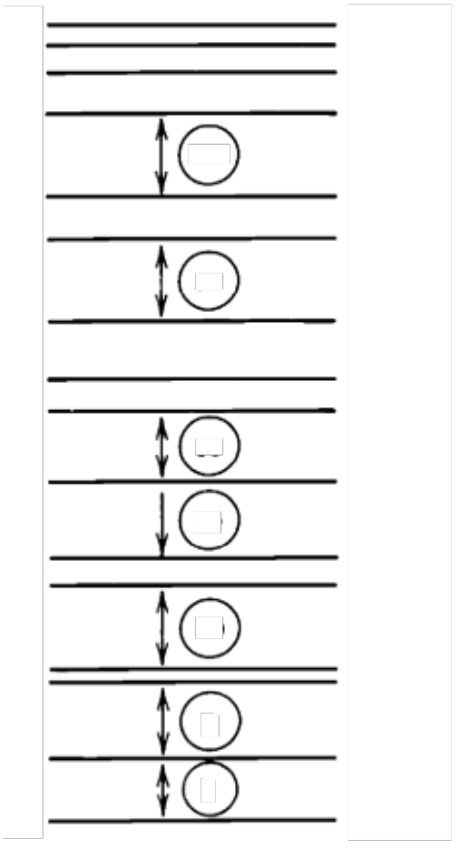
12. The image below illustrates the energy levels of a 3D spherical potential well. Based on this image, annotate the figure alongside it by indicating:

a) the energy-level labels;

b) the degeneracy (including spin);

c) the occupancy (inside the circles).





6 Nuclear Magic

13. Nuclear **magic numbers**:

a) Write down the observed magic numbers.

b) What is the significance of the magic numbers?

c) Do you notice any pattern in the magic numbers?

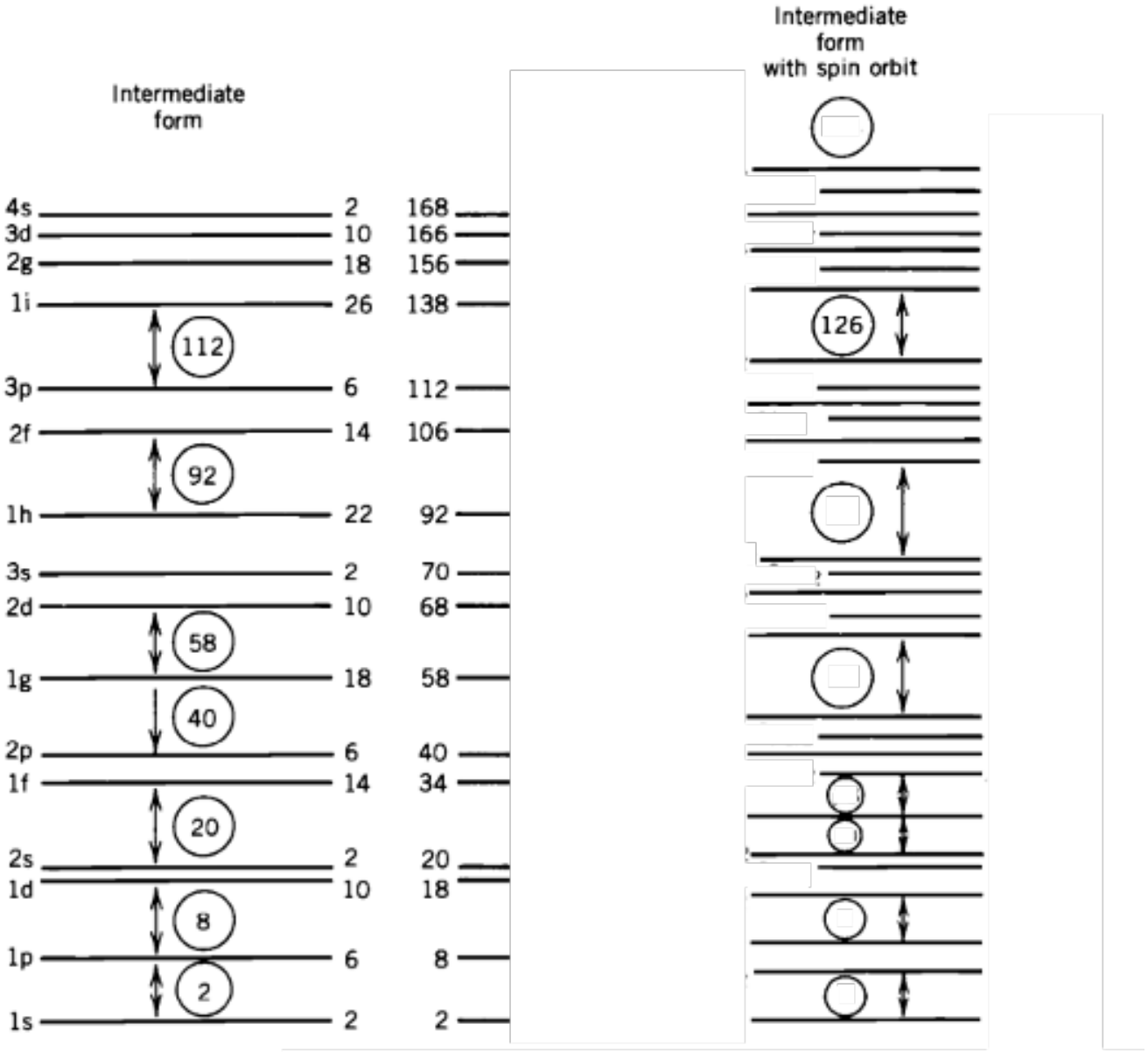
7 Woods–Saxon Potential

14. The image below shows the energy levels of the Woods–Saxon potential without (left) and with (right) the spin–orbit coupling term. Annotate the figure by indicating:

a) labels for the energy levels using the notation $1P_{1/2}$;

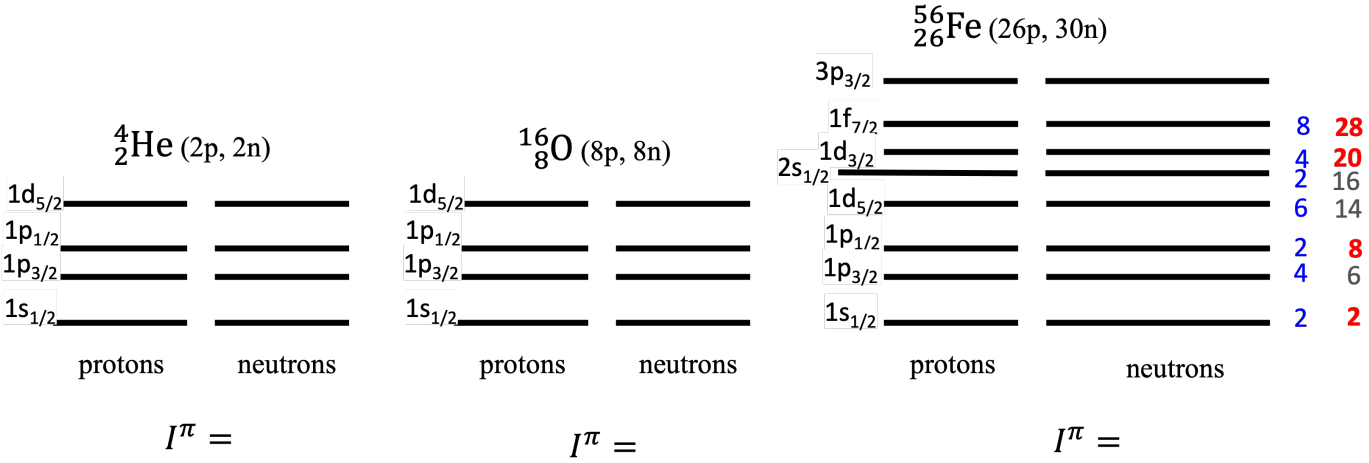
b) the degeneracy (including spin);

c) the occupancy (inside the circles).



8 Using the Shell Model

15. The images below depict the energy levels of the shell model. Determine I^π for each.



$I^\pi =$ $I^\pi =$ $I^\pi =$

16. The images below depict the energy levels of the shell model. Determine all possible values of I^π for each.

