

# Preface

This book on the *Basic Matters* of quantum mechanics grew out of a set of lecture notes for a second-year undergraduate course at the National University of Singapore (NUS). It is a first introduction that does not assume any prior knowledge of the subject. The presentation is rather detailed and does not skip intermediate steps that — as experience shows — are not so obvious for the learning student.

Starting from the simplest quantum phenomenon, the Stern–Gerlach experiment with its choice between two discrete outcomes, and ending with the standard examples of one-dimensional continuous systems, the physical concepts and notions as well as the mathematical formalism of quantum mechanics are developed in successive small steps, with scores of exercises along the way. The presentation is “modern”, a dangerous word, in the sense that the natural language of the trade — Dirac’s kets and bras and all that — is introduced early, and the temporal evolution is dealt with in a picture-free manner, with Schrödinger’s and Heisenberg’s equations of motion side by side and on equal footing.

Two companion books on *Simple Systems* and *Perturbed Evolution* cover the material of the subsequent courses at NUS for third- and fourth-year students, respectively. The three books are, however, not strictly sequential but rather independent of each other and largely self-contained. In fact, there is quite some overlap and a considerable amount of repeated material. While the repetitions send a useful message to the self-studying reader about what is more important and what is less, one could do without them and teach most of *Basic Matters*, *Simple Systems*, and *Perturbed Evolution* in a coherent two-semester course on quantum mechanics.

All three books owe their existence to the outstanding teachers, colleagues, and students from whom I learned so much. I dedicate these lectures to them.

I am grateful for the encouragement of Professors Choo Hiap Oh and Kok Khoo Phua who initiated this project. The professional help by the staff of World Scientific Publishing Co. was crucial for the completion; I acknowledge the invaluable support of Miss Ying Oi Chiew and Miss Lai Fun Kwong with particular gratitude. But nothing would have come about, were it not for the initiative and devotion of Miss Jia Li Goh who turned the original handwritten notes into electronic files that I could then edit.

I wish to thank my dear wife Ola for her continuing understanding and patience by which she is giving me the peace of mind that is the source of all achievements.

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