

Preface

This book on the *Perturbed Evolution* of quantum systems grew out of a set of lecture notes for a fourth-year undergraduate course at the National University of Singapore (NUS). The reader is expected to be familiar with the subject matter of a solid introduction to quantum mechanics, such as Dirac's formalism of kets and bras, Schrödinger's and Heisenberg's equations of motion, and the standard examples that can be treated exactly, with harmonic oscillators and hydrogen-like atoms among them.

After brief reviews of quantum kinematics and dynamics, including discussions of Bohr's principle of complementarity and Schwinger's quantum action principle, the attention turns to the elements of time-dependent perturbation theory and then to the scattering by localized interactions. Fermi's golden rule, the Born series, and the Lippmann–Schwinger equation are returning themes.

A chapter on general angular momentum prepares the ground for a discussion of indistinguishable particles. The scattering of two particles of the same kind, the basic properties of two-electron atoms, and a glimpse at many-electron atoms illustrate the matter. Throughout the text, the learning student will benefit from the dozens of exercises on the way and the detailed exposition that does not skip intermediate steps.

Two companion books on *Basic Matters* and *Simple Systems* cover the material of the preceding courses at NUS for second- and third-year students, respectively. The three books are, however, not strictly sequential but rather independent of each other and largely self-contained. In fact, there is quite some overlap and a considerable amount of repeated material. While the repetitions send a useful message to the self-studying reader about what is more important and what is less, one could do without them and teach most of *Basic Matters*, *Simple Systems*, and *Perturbed Evolution* in a coherent two-semester course on quantum mechanics.

All three books owe their existence to the outstanding teachers, colleagues, and students from whom I learned so much. I dedicate these lectures to them.

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I wish to thank my dear wife Ola for her continuing understanding and patience by which she is giving me the peace of mind that is the source of all achievements.

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Contents

Preface	vii
1. Basics of Kinematics and Dynamics	1
1.1 Brief review of basic kinematics	1
1.2 Bohr's principle of complementarity	7
1.2.1 Complementary observables	7
1.2.2 Algebraic completeness	13
1.2.3 Bohr's principle. Technical formulation	17
1.2.4 Composite degrees of freedom	17
1.2.5 The limit $N \rightarrow \infty$. Symmetric case	19
1.2.6 The limit $N \rightarrow \infty$. Asymmetric case	26
1.2.7 Bohr's principle. Quantum indeterminism	30
1.3 Brief review of basic dynamics	31
1.3.1 Equations of motion	31
1.3.2 Time transformation functions	33
1.4 Schwinger's quantum action principle	36
1.4.1 An example: Constant force	39
1.4.2 Insertion: Varying an exponential function	41
1.4.3 Time-independent Hamilton operator	43
2. Time-Dependent Perturbations	45
2.1 Born series	45
2.2 Scattering operator	48
2.3 Dyson series	50
2.4 Fermi's golden rule	52
2.5 Photon emission by a "two-level atom"	57

2.5.1	Golden-rule treatment	57
2.5.2	A more detailed treatment	60
2.5.3	An exact treatment	65
2.6	Driven two-level atom	68
2.6.1	Schrödinger equation	68
2.6.2	Resonant drive	71
2.6.3	Periodic drive	72
2.6.4	Very slow drive. Adiabatic evolution	74
2.7	Adiabatic population transfer	78
2.8	Equation of motion for the unitary evolution operator	81
3.	Scattering	87
3.1	Probability density, probability current density	87
3.2	One-dimensional prelude: Forces scatter	91
3.3	Scattering by a localized potential	95
3.3.1	Golden-rule approximation	95
3.3.2	Example: Yukawa potential	99
3.3.3	Rutherford cross section as a limit	101
3.4	Lippmann–Schwinger equation	102
3.4.1	Born approximation	110
3.4.2	Transition operator	111
3.4.3	Optical theorem	113
3.4.4	Example of an exact solution	116
3.5	Partial waves	117
3.6	s -wave scattering	122
4.	Angular Momentum	129
4.1	Spin	129
4.2	Addition of two angular momenta	134
4.2.1	Two spin- $\frac{1}{2}$ systems	137
4.2.2	Total angular momentum of an electron	138
5.	External Magnetic Field	141
5.1	Electric charge in a magnetic field	141
5.2	Electron in a homogeneous magnetic field	148
6.	Indistinguishable Particles	155
6.1	Indistinguishability	155

6.2	Bosons and fermions	157
6.3	Scattering of two indistinguishable particles	161
6.4	Two-electron atoms	165
6.4.1	Variational estimate for the ground state	165
6.4.2	Perturbative estimate for the first excited state	171
6.4.3	Self-consistent single-electron wave functions	173
6.5	A glimpse at many-electron atoms	175
	Index	181