Solutions

(1a) The Heisenberg equations of motion are here

$$\frac{d}{dt} P = -\frac{\partial H}{\partial X} = -\gamma P \quad , \quad \frac{d}{dt} X = \frac{\partial H}{\partial P} = \frac{1}{M} P + \gamma X$$

and are solved by

$$P(t) = e^{-\gamma T} P(t_0) ,$$

$$X(t) = e^{\gamma T} X(t_0) + \frac{e^{\gamma T} - e^{-\gamma T}}{2\gamma M} P(t_0)$$

with $T \equiv t - t_0$.

(1b) We have

$$[X(t), P(t_0)] = [e^{\gamma T} X(t_0), P(t_0)] = i\hbar e^{\gamma T}.$$

(1c) Proceeding from

$$i\hbar \frac{\partial}{\partial t} \langle x,t|p,t_0\rangle = \langle x,t| H_t |p,t_0\rangle$$

we express H_t in terms of $X(t)$ and $P(t_0)$. This gives

$$H = \frac{1}{2M} e^{-2\gamma T} P(t_0)^2 + \frac{1}{2}\gamma e^{-\gamma T}\big(X(t)P(t_0) + P(t_0)X(t)\big)$$

$$= \frac{1}{2M} e^{-2\gamma T} P(t_0)^2 + \gamma e^{-\gamma T} X(t)P(t_0) - \frac{1}{2} i\hbar\gamma$$

so that

$$i\hbar \frac{\partial}{\partial t} \langle x,t|p,t_0\rangle = \left(\frac{1}{2M} e^{-2\gamma T} p^2 + \gamma e^{-\gamma T} xp - \frac{1}{2} i\hbar\gamma\right)\langle x,t|p,t_0\rangle$$

$$= \langle x,t|p,t_0\rangle \frac{\partial}{\partial t}\left(\frac{p^2}{2M}\,\frac{1-e^{-2\gamma T}}{2\gamma} + xp(1-e^{-\gamma T}) - \frac{1}{2} i\hbar\gamma T\right).$$