Question Test 1/1

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1 (a) $S = \frac{3}{7}\frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} + \frac{4}{7}\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} = \frac{1}{14}\begin{pmatrix}11 & 3\\ 3 & 3\end{pmatrix}$

(b) $S = \frac{1}{7}\frac{1}{10}\begin{pmatrix}1 & -3\\ -3 & 9\end{pmatrix} + \frac{6}{7}\frac{1}{10}\begin{pmatrix}9 & 3\\ 3 & 1\end{pmatrix} = \frac{1}{70}\begin{pmatrix}55 & 15\\ 15 & 15\end{pmatrix}$

$= \frac{1}{14}\begin{pmatrix}11 & 3\\ 3 & 3\end{pmatrix}$

(c) $S = \frac{12}{77}\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix} + \frac{65}{77}\frac{1}{130}\begin{pmatrix}121 & 33\\ 33 & 9\end{pmatrix}$

$= \frac{1}{11 \cdot 14}\begin{pmatrix}121 & 33\\ 33 & 33\end{pmatrix} = \frac{1}{14}\begin{pmatrix}11 & 3\\ 3 & 3\end{pmatrix}$

It is the same mixture for all three blends.

2 (a) The eigenvalues a obey the same equation as the operator A: $a^2 = a$, so that $a=0$ and $a=1$ are the only possible values.

(b) All higher powers of A are equal to A itself. Therefore, $f(A) = f_0 + f_1 A$ with $f(0) = f_0$, $f(1) = f_0 + f_1$, or $f_0 = f(0)$, $f_1 = f(1) - f(0)$.

(c) Here $f(0) = \cos(\pi \cdot 0) = 1$, $f(1) = \cos(\pi \cdot 1) = -1$, so that $f_0 = 1$, $f_1 = -2$, and

$$\cos(\pi A) = 1 - 2A.$$

3 Since

$$e^{i\lambda P^3/\hbar} e^{-\mu X} e^{-i\lambda P^3/\hbar}$$

$$= e^{-\mu\left(e^{i\lambda P^3/\hbar} X e^{-i\lambda P^3/\hbar}\right)}$$

and

$$e^{i\lambda P^3/\hbar} X e^{-i\lambda P^3/\hbar} = X + 3\lambda P^2,$$

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we have $e^{i\lambda P^3/\hbar} e^{-\mu X} e^{-i\lambda P^3/\hbar} = e^{-(\mu X + 3\lambda P^2)}$

$= e^{-\mu X - 3\lambda P^2}$, indeed.

$\boxed{4}$ (a)

$$\langle X \rangle = \int dp \langle\ |p\rangle\langle p|X|\ \rangle = \int dp\, \psi(p)^* i\hbar \frac{\partial}{\partial p}\psi(x),$$

$$\langle X^2 \rangle = \int dp \langle\ |X|p\rangle\langle p|X|\ \rangle$$

$$= \int dp \left(-i\hbar \frac{\partial \psi^*}{\partial p}\right)\left(i\hbar \frac{\partial \psi}{\partial p}\right)$$

$$= \int dp \left|\hbar \frac{\partial}{\partial p}\psi(p)\right|^2,$$

$$\langle PX \rangle = \int dp \langle\ |P|p\rangle\langle p|X|\ \rangle$$

$$= \int dp\, p\, \psi^*(p)\, i\hbar \frac{\partial}{\partial p}\psi(p).$$

(b) $\frac{\partial}{\partial p}\psi(p) = 2\sqrt{a^3}\,(1-ap)e^{-ap}$ for $p>0$ and $=0$ for $p<0$.

$\langle X \rangle = 0$ since $\psi(p)$ is real, or explicitly

$$\langle X \rangle = 4a^3 \int_0^\infty dp\, p(1-ap)\, e^{-2ap}$$

$$= 4a^3\left(\frac{1!}{(2a)^2} - \frac{2!\,a}{(2a)^3}\right) = 0.$$

$$\langle X^2 \rangle = 4a^3\hbar^2 \int_0^\infty dp\, (1-ap)^2 e^{-2ap}$$

$$= 4a^3\hbar^2\left(\frac{1!}{2a} - 2\frac{2!\,a}{(2a)^2} + \frac{2!\,a^2}{(2a)^3}\right)$$

$$= (\hbar a)^2.$$

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$$\langle PX \rangle = 4a^3 i\hbar \int_0^\infty dp\, p^2(1-ap)\, e^{-2ap}$$

$$= 4a^2 i\hbar \left(\frac{2!}{(2a)^3} - \frac{3!\,a}{(4a)^4} \right) = -\tfrac{1}{2} i\hbar.$$

5 (a) $$\frac{\langle x|R|p\rangle}{\langle x|p\rangle} = \frac{2\langle -x|p\rangle}{\langle x|p\rangle} = \frac{2e^{-ixp/\hbar}}{e^{ixp/\hbar}}$$

$$= 2e^{-2ixp/\hbar}$$

so that $$R = 2e^{-2i X_i P/\hbar}.$$

(b) $$\mathrm{tr}\{R\} = \int \frac{dx\,dp}{2\pi\hbar}\, 2e^{-2ixp/\hbar}$$

$$= \int dx\, 2\,\delta(2x) = 1.$$