Sample Solution to Problem 1
For $n \neq 0$ you get

$$f_n = \int_{-T/2}^{T/2} \frac{\mathrm{d}t}{T}\, \mathrm{e}^{\mathrm{i}n\omega t}\, F(t) = \int_{-T/2}^{T/2} \frac{\mathrm{d}t}{T}\, \mathrm{e}^{\mathrm{i}n\omega t} \left(\frac{2t}{T}\right)^2 = (-1)^n \frac{8}{(n\omega T)^2} = (-1)^n \frac{2}{(n\pi)^2}$$

by the integration method of your choice. For $n = 0$ you get

$$f_0 = \int_{-T/2}^{T/2} \frac{\mathrm{d}t}{T}\, F(t) = \int_{-T/2}^{T/2} \frac{\mathrm{d}t}{T} \left(\frac{2t}{T}\right)^2 = \frac{1}{3}\,.$$

Accordingly,

$$F(t) = \frac{1}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos(n\omega t)\,,$$

which gives

$$1 = \frac{1}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} (-1)^n \quad \text{or} \quad \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

for $t = T/2$, that is $\omega t = \pi$, and

$$0 = \frac{1}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \quad \text{or} \quad \sum_{n=1}^{\infty} \frac{(-1)^2}{n^2} = -\frac{\pi^2}{12}$$

for $t = 0$.

Sample Solution to Problem 2
We know that

$$\begin{aligned} G(t) &= \int \frac{\mathrm{d}\omega}{2\pi}\, \mathrm{e}^{-\mathrm{i}\omega t} g(\omega) \\ = G(t)^* &= \int \frac{\mathrm{d}\omega}{2\pi}\, \mathrm{e}^{\mathrm{i}\omega t} g(\omega)^* = \int \frac{\mathrm{d}\omega}{2\pi}\, \mathrm{e}^{-\mathrm{i}\omega t} g(-\omega)^* \\ = G(-t) &= \int \frac{\mathrm{d}\omega}{2\pi}\, \mathrm{e}^{\mathrm{i}\omega t} g(\omega) = \int \frac{\mathrm{d}\omega}{2\pi}\, \mathrm{e}^{-\mathrm{i}\omega t} g(-\omega) \end{aligned}$$

so that $g(\omega) = g(-\omega)^* = g(-\omega)$, that is: $g(\omega)$ is even and real.

Sample Solution to Problem 3
Simply

$$\mathbf{x} = \frac{\mathbf{b}\times\mathbf{c}}{\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})}\,,$$

as one verifies immediately by inspection.

Sample Solution to Problem 4

(a) $\nabla\cdot\mathbf{B} = (\mathbf{b}\cdot\mathbf{r})^2\,\nabla\cdot\mathbf{r} + \mathbf{r}\cdot\nabla(\mathbf{b}\cdot\mathbf{r})^2 = 3(\mathbf{b}\cdot\mathbf{r})^2 + \mathbf{r}\cdot[2\mathbf{b}\,(\mathbf{b}\cdot\mathbf{r})] = 5(\mathbf{b}\cdot\mathbf{r})^2\,.$

(b) $\nabla\times\mathbf{B} = (\mathbf{b}\cdot\mathbf{r})^2\,\nabla\times\mathbf{r} + [\nabla(\mathbf{b}\cdot\mathbf{r})^2]\times\mathbf{r} = 0 + [2\mathbf{b}\,(\mathbf{b}\cdot\mathbf{r})]\times\mathbf{r} = 2(\mathbf{b}\cdot\mathbf{r})\mathbf{b}\times\mathbf{r}\,.$

(c) Take $\mathbf{b} = b\mathbf{e}_z$, which is to say that we orient the coordinate system such that the z axis coincides with the direction of $\mathbf{b}$. Then

$$\begin{aligned}
\int_{S_R} \mathrm{d}\mathbf{S}\cdot\mathbf{B} &= \int \mathrm{d}\Omega\, r^3 b^2 r^2 (\cos\vartheta)^2\Big|_{r=R} \\
&= 2\pi R^5 b^2 \int_0^\pi \mathrm{d}\vartheta\,\sin\vartheta\,(\cos\vartheta)^2 = \frac{4\pi}{3}R^5 b^3 \\
= \int_{V_R} (\mathrm{d}\mathbf{r})\,\nabla\cdot\mathbf{B} &= \int_0^R \mathrm{d}r\, r^2\, 2\pi \int_0^\pi \mathrm{d}\vartheta\,\sin\vartheta\, 5b^2 r^2 (\cos\vartheta)^2 \\
&= \int_0^R \mathrm{d}r\, r^4\, 2\pi 5 \int_0^\pi \mathrm{d}\vartheta\,\sin\vartheta\,(\cos\vartheta)^2 \\
&= R^5\, 2\pi\,\frac{2}{3} = \frac{4\pi}{3}R^5 b^3\,.
\end{aligned}$$

Sample Solution to Problem 5

$\nabla\cdot[a(\mathbf{r})\mathbf{B}(\mathbf{r})\times\mathbf{C}(\mathbf{r})] = (\mathbf{B}\times\mathbf{C})\cdot\nabla a + a\mathbf{C}\cdot(\nabla\times\mathbf{B}) - a\mathbf{B}\cdot(\nabla\times\mathbf{C})\,.$