Problem 1 (25 marks)

The periodic function $F(t) = F(t+T) = \sum_{n=-\infty}^{\infty} f_n \mathrm{e}^{-\mathrm{i}n\omega t}$ with $\omega T = 2\pi$ is specified by

$$F(t) = \left(\frac{2t}{T}\right)^2 \quad \text{for} \quad -\frac{T}{2} \le t \le \frac{T}{2}\,.$$

Determine the Fourier coefficients f_n. Then consider $t = 0$ and $t = T/2$ to re-derive the values of two well-known series. [Hint: Remember about the $n = 0$ term.]

Problem 2 (15 marks)

The function $G(t) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi}\, \mathrm{e}^{-\mathrm{i}\omega t}\, g(\omega)$ is known to be real and even,

$$G(t) = G(t)^* = G(-t)\,.$$

What are the corresponding properties of $g(\omega)$ that are implied by these properties of $G(t)$?

Problem 3 (15 marks)

The three vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ do not lie in the same plane, so that $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) \neq 0$. Determine the vector $\mathbf{x}$ that obeys

$$\mathbf{a} \cdot \mathbf{x} = 1\,, \qquad \mathbf{b} \cdot \mathbf{x} = 0\,, \qquad \mathbf{c} \cdot \mathbf{x} = 0\,.$$

[Hint: The answer is simple.]

Problem 4 (25 marks)

Vector field $\mathbf{B}(\mathbf{r})$ is given by $\mathbf{B}(\mathbf{r}) = (\mathbf{b} \cdot \mathbf{r})^2 \mathbf{r}$, whereby $\mathbf{b}$ is a constant vector that does not depend on the position vector $\mathbf{r}$.

- **(a)** Determine the divergence of $\mathbf{B}(\mathbf{r})$.
- **(b)** Determine the curl of $\mathbf{B}(\mathbf{r})$.
- **(c)** Evaluate the surface integral

$$\int_{S_R} \mathrm{d}\mathbf{S} \cdot \mathbf{B}$$

whereby S_R is the sphere of radius R, centered at $\mathbf{r} = 0$.
[Hint: Choose a convenient direction for $\mathbf{b}$.]

Problem 5 (20 marks)

$a(\mathbf{r})$ is a scalar field; $\mathbf{B}(\mathbf{r})$ and $\mathbf{C}(\mathbf{r})$ are vector fields. Express

$$\nabla \cdot [a(\mathbf{r})\mathbf{B}(\mathbf{r}) \times \mathbf{C}(\mathbf{r})]$$

as a sum of three terms, such that only $a(\mathbf{r})$ is differentiated in the first term, only $\mathbf{B}(\mathbf{r})$ in the second term, and only $\mathbf{C}(\mathbf{r})$ in the third term.