Problem 1 (20 marks)
A harmonic oscillator is in the coherent state described by the ket $|a\rangle$. Determine the expectation values of position X and momentum P and their spreads δX and δP. How large is their product $\delta X\,\delta P$?

Problem 2 (20 marks)
Orbital angular momentum: If the system is in an eigenstate of $\vec{L}^2$ with eigenvalue $2\hbar^2$, what are the possible outcomes when a measurement of $L_1L_2 + L_2L_1$ is performed?

Problem 3 (30 marks)
A harmonic oscillator (natural frequency ω, ladder operators A and $A^\dagger$) is perturbed by a potential proportional to $\mathrm{i}({A^\dagger}^2 - A^2)$, so that the Hamilton operator is

$$H = \hbar\omega A^\dagger A + \mathrm{i}\hbar\Omega({A^\dagger}^2 - A^2) \qquad \text{with } |\Omega| < \tfrac{1}{2}\omega\,.$$

Introduce new ladder operators B and $B^\dagger$ as linear combinations of A and $A^\dagger$ (that is $B = \alpha A + \beta A^\dagger$ with $[B, B^\dagger] = 1$, of course), such that

$$H = \hbar\omega' B^\dagger B + E_0$$

and determine the ground state energy of E_0 thereby.
[Hint: You'll need to establish three equations for $|\alpha|$, $|\beta|$, and ω'.]

Problem 4 (30 marks)
Motion along the x axis; position operator X, momentum operator P. The ground state energy E_0 of the Hamilton operator

$$H = \frac{P^2}{2M} + \frac{1}{2}M\omega^2X^2 + F|X| \qquad \text{with } M > 0,\ \omega > 0,\ F \text{ arbitrary}$$

is a function of the parameters M, ω, and F. Determine $\left.\dfrac{\partial E_0}{\partial F}\right|_{F=0}$.